Capital Goods Prices, R&D, and Long-Run Growth: The Declining Labor Share in a Revisited Romer Model

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Abstract

This paper revisits the Romer (1990) endogenous growth model to examine the joint determination of long-run growth, R&D intensity, and the functional distribution of income in an economy with declining capital goods prices. We introduce investment-specific technological change and model R&D as a capital-intensive activity, generating an endogenous trade-off in resource allocation. The balanced-growth equilibrium features an additively separable growth structure, combining innovation-driven growth with capital-efficiency gains. We show that a slowdown in capital-embodied technological progress reallocates resources toward R&D while reducing long-run economic growth, offering a theoretical explanation for the innovation paradox. Moreover, when R&D activities are capitalized in the measurement of GDP, the labor share declines even under a Cobb-Douglas production structure.

Keywords Endogenous Growth; Research and Development; Investment-Specific Technological Change; Capital Goods Prices; Labor Share

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1 Introduction

Over the past decades, advanced economies have exhibited a striking constellation of empirical trends. Measured labor shares have declined (Karabarbounis and Neiman, 2014; Autor et al., 2020), the relative price of investment goods has fallen substantially (Greenwood et al., 1997; Jorgenson, 2001), and R&D expenditures have risen steadily as a share of GDP (Jones, 2015; OECD, 2022). At the same time, aggregate productivity growth has slowed, often discussed under the rubric of secular stagnation (Summers, 2014).

These developments are closely intertwined. The persistent decline—and more recently the flattening—in the relative price of capital goods (Byrne and Pinto, 2015; Byrne et al., 2016) reflects changes in investment-specific technological progress, which in turn shape capital accumulation and resource allocation. Yet, despite intensifying R&D efforts, productivity growth has remained subdued, a pattern sometimes described as an " innovation paradox " (Aghion et al., 2021). Recent work also emphasizes that the changing nature of innovation may reshape the functional distribution of income (Acemoglu, 2024), further underscoring the importance of understanding how technological change interacts with resource allocation.

Together, these trends raise a deeper question: how does investment-specific technological change reshape the allocation of resources between production and innovation, and what are the implications for long-run growth and income distribution? More specifically, can a unified framework account simultaneously for (i) the stagnation of long-run growth, (ii) the decline in measured labor shares, (iii) the sustained decline?and recent flattening?of the relative price of capital goods, and (iv) the rising intensity of R&D expenditure?

We argue that these phenomena can be understood as the joint outcome of investment-specific technological change and equilibrium resource reallocation within an endogenous growth framework. To formalize this mechanism, we revisit the Romer (1990) model not by abandoning its core structure, but by embedding the embodied technological progress emphasized by Greenwood, Hercowitz, and Krusell (1997) into its innovation structure. By integrating endogenous innovation with capital-embodied technological change, our framework provides a unified structure in which changes in capital efficiency reshape resource allocation, long-run growth, and income distribution, while preserving analytical tractability and a well-behaved balanced-growth equilibrium.

This structure generates a transparent set of equilibrium mechanisms. Because capital is used both in production and in innovation activities, changes in the efficiency of capital goods affect the equilibrium allocation of resources between final-good production and R&D. In equilibrium, aggregate growth can be decomposed into two distinct components: one reflecting the direct efficiency gains from capital-embodied technological change, and another arising from its impact on innovation incentives through resource reallocation. Remarkably, these two forces can be cleanly separated, yielding a transparent growth decomposition.

A key implication of the model is that a slowdown in investment-specific technological progress can increase R&D intensity while reducing aggregate growth. The mechanism operates through equilibrium reallocation rather than through exogenous changes in research productivity. In this sense, R&D intensity is not a sufficient statistic for innovation-driven growth. At the same time, because GDP is defined as value added rather than gross output, the shift toward capital-intensive R&D activities leads to a decline in measured labor shares even though factor shares in the underlying production technology remain constant.

To preserve a well-behaved balanced-growth path in the presence of capital-augmenting technological change, we adopt a Cobb-Douglas production structure. This choice is motivated not by a claim about the precise value of the elasticity of substitution, but by the need to maintain a stable price system and a constant interest rate along the balanced-growth path. The Cobb-Douglas specification allows investment-specific technological change and endogenous innovation to coexist without generating explosive or nonstationary factor price dynamics. This structural feature allows us to derive clear empirical and theoretical implications for growth, R&D dynamics, and income distribution.

This paper makes three main contributions.

First, it provides a unified endogenous growth framework that links investment-specific technological change to resource reallocation between production and innovation. We show that a slowdown in capital-embodied technological progress can simultaneously generate rising R&D intensity and declining aggregate growth, offering a structural explanation for the observed innovation paradox.

Second, the model demonstrates how changes in investment-specific technological progress and R&D composition affect measured labor shares. Even within a Cobb?Douglas production structure, the expansion of value added through capital-intensive R&D can lead to a decline in measured labor shares while preserving constant factor shares in gross production.

Third, we demonstrate that the core growth mechanism is robust to alternative specifications of R&D inputs. Introducing human capital explicitly into innovation preserves the growth decomposition and the allocation channel, while clarifying how the composition of R&D influences measured labor shares. This robustness underscores the structural nature of our results.

The remainder of this paper is organized as follows. Section 2 presents the model and characterizes the static equilibrium. Section 3 analyzes the balanced-growth equilibrium and derives the main results on growth and resource allocation. Section 4 examines the implications for GDP measurement and factor income shares. Section 5 extends the model to incorporate human capital. Section 6 concludes.

2 The model

The model builds on the endogenous technological change framework of Romer (1990). While the structure of the R&D sector follows Romer 's variety-expansion mechanism, we modify the production side to incorporate capital-embodied technological change in a tractable manner. In particular, we adopt a production structure that preserves a balanced growth path with a constant interest rate, allowing investment-specific technological progress to interact with endogenous innovation without destabilizing factor prices.

The economy consists of four sectors: final goods, capital goods, intermediate goods, and R&D. There are three fundamental state variables: physical capital (K), raw labor (L), and knowledge (A), where A denotes the measure of available intermediate varieties. Time is continuous, and the final good serves as the num'eraire.

Following Matsuyama (1999), capital is interpreted broadly to include both physical capital and knowledge-embedded assets. This composite capital input is used both in intermediate production and in research activities. Final goods are produced using raw labor and a continuum of differentiated intermediate inputs indexed by $i \in [0, A]$. The R&D sector expands the set of available varieties by combining existing knowledge with capital inputs.

The final goods sector Final goods Y are produced using labor input L and a continuum of differentiated intermediate goods indexed by $z \in [0, A]$:

$$Y = L^{1-\alpha} \int_0^A X(z)^\alpha \, dz, \quad \alpha \in (0, 1). \tag{1}$$

Here, α is the output elasticity of each intermediate good, assumed to be constant and exogenously given.

Cost minimization in the final goods sector yields the wage and the inverse demand for intermediate goods:

$$w = (1-\alpha)\frac{Y}{L}, \quad \text{and} \quad p(z) = \alpha L^{1-\alpha} X(z)^{\alpha-1}. \tag{2}$$

Final goods are assumed to be used for consumption or investment, so that the resource constraint is given by

$$Y = C + \dot{K}.$$

The capital goods sector This section introduces a competitive capital goods sector that supplies capital inputs to intermediate goods producers. Capital goods are produced by renting durable capital $\tilde{K}$ from the capital market at rental rate r. Combining η units of durable capital yields one unit of capital good $\mathcal{K}$ according to the linear technology

$$\mathcal{K} = \frac{1}{\eta}\tilde{K}.$$

A representative capital goods producer solves

$$\max_{\tilde{K}} \ \Pi^K = p_{\mathcal{K}}\mathcal{K} - r\tilde{K},$$

where $p_{\mathcal{K}}$ denotes the price of capital goods. Given perfect competition and the linear production technology, the price of capital goods equals marginal cost,

$$p_{\mathcal{K}} = \eta r.$$

To capture exogenous improvements in the efficiency of capital goods production, we assume that η declines over time at a constant rate,

$$\frac{\dot{\eta}}{\eta} = -\gamma < 0.$$

For a given rental rate r, a decline in η implies a decreasing price of capital goods.

The intermediate goods sector Each intermediate good is supplied by a monopolist. Monopoly power is generated by a patent held by the R&D firm that develops the variety. Intermediate production uses capital goods $\mathcal{K}(z)$ as the sole input, and we normalize the intermediate production technology to be linear:

$$X(z) = \mathcal{K}(z).$$

Hence, the period profit of the monopolist producing variety z is given by

$$\tilde{\pi}(z) = p(z)X(z) - p_{\mathcal{K}}\mathcal{K}(z). \tag{3}$$

Maximizing (3) with respect to $X(z)$, taking $p(z)$ as given by (2), we obtain

$$\tilde{X} \equiv X(z) = \left[\frac{\alpha^2}{r\,\eta}\right]^{\frac{1}{1-\alpha}} L, \quad \text{and} \quad \tilde{p} = p(z) = \frac{r\,\eta}{\alpha}. \tag{4}$$

Equation (4) implies that demand is symmetric across varieties, so we can write $X(z) \equiv \tilde{X}$ and $p(z) = \tilde{p}$ for all $z \in [0, A]$. Substituting (4) into (3), we have the per-variety profit

$$\tilde{\Pi}(z) = \left(\frac{1}{\alpha} - 1\right) r\,\eta \tilde{X} \equiv \tilde{\Pi}, \tag{5}$$

where the definition of $\tilde{\Pi}$ follows from symmetry across varieties.

Aggregate capital used in intermediate production is defined as

$$K_Y = \eta \int_0^A X(z)dz = \eta \underbrace{\tilde{X}A}_{\equiv X} = \eta X, \quad \text{namely} \quad \tilde{X} = \frac{K_Y}{\eta A}. \tag{6}$$

This equation shows that η governs the efficiency with which durable capital is transformed into effective capital goods (and thus the relative price of capital goods in units of the final good, which is the numéraire). Substituting $X(z) = \tilde{X}$ and (6) into (1), we obtain the reduced-form production function:

$$Y = \eta^{-\alpha} K_Y^{\alpha} A^{1-\alpha} L^{1-\alpha}. \tag{7}$$

Combining (4), (6), and (7), we obtain the rental rate of capital:

$$r = \alpha^2 \eta^{-\alpha} K_Y^{\alpha-1} A^{1-\alpha} L^{1-\alpha} = \alpha^2 \frac{Y}{K_Y}. \tag{8}$$

Combining (6) and (8) with (5), we obtain the per-variety profit:

$$\tilde{\Pi} = (1-\alpha)\alpha \frac{Y}{A}. \tag{9}$$

R&D sector R&D activities create new varieties of intermediate goods. The value of a newly created variety, denoted by $\tilde{V}$, equals the present discounted value of the stream of monopoly profits it generates:

$$\tilde{V} = \int_0^\infty \tilde{\Pi}(s) e^{-\int_0^s r(\tau)d\tau} ds. \tag{10}$$

The discount rate $r(t)$ denotes the equilibrium interest rate. Under perfect capital markets and in the absence of arbitrage, this rate equals the rental rate of capital determined in the production sector.

Differentiating with respect to time yields the no-arbitrage condition:

$$r\tilde{V} = \tilde{\Pi} + \dot{\tilde{V}}. \tag{11}$$

Our specification differs from the traditional lab-equipment model (Rivera-Batiz and Romer, 1991), in which final goods serve as inputs into R&D. Instead, following Matsuyama (1999), we assume that capital?broadly defined?enters innovation activities.

The production function for new varieties is given by

$$\dot{A}(t) = \frac{\delta A(t) K_A(t)}{\Omega(t)}, \tag{12}$$

where $\Omega(t)$ captures the difficulty of innovation, as in Segerstrom (1998). Since the growth rate of varieties is $\dot{A}/A = \delta K_A/\Omega$, balanced growth requires that the ratio K_A/Ω remain stationary. This implies that $\Omega(t)$ must grow at the same rate as aggregate capital in equilibrium. For analytical transparency, we adopt the normalization

$$\Omega(t) = K(t),$$

which preserves balanced growth while isolating the role of capital prices in shaping innovation incentives.

Profits of the j-th R&D firm are given by

$$\Pi_j^{RD}(t) = \dot{A}_j(t)\tilde{V}(t) - r(t)K_{Aj}(t) = (\delta A(t)\tilde{V}(t) - r(t))K_{Aj}(t). \tag{13}$$

Free entry implies

$$\delta \frac{A(t)\tilde{V}(t)}{\Omega(t)} \leq r(t), \tag{14}$$

with equality if $K_A(t) > 0$. Under positive R&D activity,

$$\tilde{V} = r\frac{\Omega}{\delta A} = \alpha^2 \frac{Y\Omega}{\delta A K_Y}. \tag{15}$$

Substituting (9) and (15) into (11), we obtain

$$r = \frac{\dot{\tilde{V}}}{\tilde{V}} + \frac{(1-\alpha)\delta}{\alpha\Omega} K_Y. \tag{16}$$

Defining $u \equiv K_Y/K$, we can write

$$r = \frac{\dot{\tilde{V}}}{\tilde{V}} + \delta\frac{1-\alpha}{\alpha} u. \tag{17}$$

3 Balanced-Growth Equilibrium

This section characterizes the balanced-growth equilibrium (BGP) of the model. Along the BGP, aggregate variables grow at constant rates, while appropriately detrended variables remain stationary. To isolate the structural mechanisms governing long-run growth and income distribution, we abstract from transitional dynamics and focus exclusively on BGP properties.

Let $g_Z \equiv \dot{Z}/Z$ denote the growth rate of variable Z. Along the BGP, we denote by Z^* the stationary (detrended) value of Z.

Along the BGP, the aggregate resource constraint $\dot{K} = Y - C$ implies

$$g_K^* = g_Y^* = g_C^* \equiv g^*.$$

Define the allocation of capital as $u \equiv K_Y/K$, and denote its BGP value by u^*.

Detrended variables and factor prices

To facilitate the BGP analysis, define

$$\tilde{y} = \frac{Y}{\eta^{\frac{\alpha}{1-\alpha}}AL}, \quad \tilde{k} = \frac{K}{\eta^{\frac{\alpha}{1-\alpha}}AL}, \quad \tilde{\pi} = \frac{\Pi}{\eta^{\frac{\alpha}{1-\alpha}}AL}.$$

Using (7), (8), and (9),

$$\tilde{y} = \tilde{k}^\alpha u^\alpha, \quad r = \alpha^2 \tilde{k}^{\alpha-1} u^{\alpha-1}, \quad \tilde{\pi} = (1-\alpha)\alpha\tilde{k}^\alpha u^\alpha.$$

Let

$$\tilde{v} = \frac{\tilde{V}}{\eta^{\frac{\alpha}{1-\alpha}}L}.$$

Using (15), (17), and $g_{\tilde{V}} = g_{\tilde{v}} + \frac{\alpha}{1-\alpha}\gamma$, we obtain

$$\tilde{v} = \frac{\alpha^2}{\delta}\tilde{k}^\alpha u^{\alpha-1}, \quad r = \frac{\dot{\tilde{v}}}{\tilde{v}} + \frac{\alpha}{1-\alpha}\gamma + n + \frac{1-\alpha}{\alpha}\delta u. \tag{18}$$

Household Let $c(t) \equiv C(t)/L(t)$ denote per-capita consumption. The representative household maximizes

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t}\, dt, \quad \rho > 0,\ \theta > 0.$$

The Euler equation is

$$\rho + \theta\frac{\dot{c}}{c} = r - n. \tag{19}$$

The transversality condition is

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)a(t) = 0,$$

where $a(t)$ denotes per-capita asset holdings and $\lambda(t)$ is the associated co-state variable.

Equilibrium allocation

Combining (18) and (19), the BGP interest rate satisfies

$$\theta\left[\delta(1-u^*) + \frac{\alpha}{1-\alpha}\gamma\right] = r^* - n - \rho, \tag{20}$$

$$r^* = n + \frac{\alpha}{1-\alpha}\gamma + \frac{1-\alpha}{\alpha}\delta u^*. \tag{21}$$

Eliminating r^* yields

$$u^* = \frac{\theta\delta - (1-\theta)\frac{\alpha}{1-\alpha}\gamma + \rho}{\delta\left(\frac{1-\alpha}{\alpha} + \theta\right)}.$$

An interior equilibrium requires $u^* \in (0,1)$, implying that R&D profitability must be sufficiently strong relative to impatience.

Higher R&D efficiency (δ) strengthens innovation incentives, whereas greater impatience (higher ρ) weakens them. In contrast, the effect of investment-specific technological progress (γ) depends critically on intertemporal preferences. The sign of its impact on u^* is governed by θ, with $\theta = 1$ constituting a sharp threshold.

Proposition 1 *If $\theta > 1$, an increase in γ raises u^*, reallocating capital toward intermediate production and away from R&D.*

If $\theta < 1$, an increase in γ lowers u^*, shifting capital toward R&D.

This reallocation channel links capital-price dynamics to innovation incentives.

Long-run growth

The long-run growth rate is

$$g^* = \frac{\frac{1-\alpha}{\alpha}\delta - \rho}{\frac{1-\alpha}{\alpha} + \theta} + \frac{\frac{1}{1-\alpha}\gamma}{\frac{1-\alpha}{\alpha} + \theta} + n.$$

Growth reflects both innovation incentives and capital-efficiency gains. Investment-specific technological progress affects growth directly through capital efficiency and indirectly through equilibrium allocation.

Result 1 *A decline in investment-specific technological progress (γ) lowers u^* under $\theta > 1$, reallocating capital toward R&D. However, this reallocation cannot fully compensate for the direct loss of capital efficiency. Consequently, the long-run growth rate declines despite an increase in R&D intensity.*

Thus, higher R&D intensity is not sufficient to sustain growth when capital-efficiency gains weaken.

Discussion The model provides a structural explanation for how changes in capital-price dynamics reshape innovation incentives and long-run growth. A slowdown in γ weakens the capital-efficiency component of growth and induces endogenous reallocation toward R&D when $\theta > 1$. Yet slower growth emerges from equilibrium interactions rather than from declining research productivity.

This mechanism is consistent with the documented flattening in the decline of relative capital prices (Byrne and Pinto, 2015; Byrne et al., 2016). Within the model, such a flattening corresponds to a reduction in γ, triggering higher R&D intensity alongside slower aggregate growth.

The framework therefore offers a purely structural account of the coexistence of rising R&D intensity and declining long-run growth.

4 GDP and Labor Share

This section analyzes the steady-state functional distribution of GDP. Throughout this section, we restrict attention to the steady state. For notational simplicity, the steady-state R&D expenditure share u^* is denoted by u. GDP (gross domestic product) is defined as the aggregate value added produced within an economy. It is distributed between labor income and capital income, where the latter consists of rental payments on capital goods and monopoly profits generated in the intermediate sector.

Then, the gross domestic product, denoted by Υ (which differs from final goods production Y), can be expressed as

$$\Upsilon = wL + rK + \Pi.$$

The production of new ideas through R&D activities generates monopoly profits (Π), which we treat as a component of capital income in gross terms. This treatment aligns with the 2008 System of National Accounts (United Nations, 2009), where R&D expenditures are capitalized as fixed assets rather than being recorded as intermediate consumption.

Since the model adopts a Cobb-Douglas production structure in the spirit of Romer (1990), factor income shares relative to final goods production remain constant:

$$s_L = \frac{wL}{Y} = 1-\alpha, \quad s_{K_Y} = \frac{rK_Y}{Y} = \alpha^2, \quad \text{and} \quad s_\Pi = \frac{\Pi}{Y} = (1-\alpha)\alpha$$

where s_J ($J = L, K_A, \Pi$) denotes the share of output (final goods production) allocated to each component. The condition $s_L + s_{K_Y} + s_\Pi = 1$ implies that the total output Y is exhaustively distributed among these three factors (i.e., $Y = wL + rK_Y + \Pi$). The key discrepancy between Υ and Y stems from whether the factor payment rK_A is included. For this reason, we incorporate the value added generated by successful R&D activities into the definition of GDP.

By including the value created in the R&D sector in the aggregate GDP (defined as Υ), which serves as the denominator for the distribution shares, the total value-added is exhaustively distributed among K, L, and Π. Consequently, Υ is derived as a function of Y and u as follows:

$$\begin{aligned}
\Upsilon &= rK + wL + \Pi \\
&= Y + \dot{A}\tilde{V} = Y + \frac{\delta A K_A}{\Omega}\frac{r\Omega}{\delta A} \\
&= Y + rK_A = Y + \alpha^2 \frac{Y}{u}(1-u) \\
&= \left[1 + \frac{\alpha^2(1-u)}{u}\right] Y
\end{aligned}$$

Using the derived expression $\Upsilon = \left[1 + \frac{\alpha^2(1-u)}{u}\right] Y$, one can confirm that the equivalence between the distribution and production sides of the economy is satisfied (i.e., $\Upsilon = wL + \underbrace{rK}_{=rK_Y + rK_A} + \Pi$). It should be noted that the long-run economic growth rate (GDP growth rate) coincides with the growth rate of Y, since Υ differs from Y only by a stationary multiplicative factor in balanced growth.It should be noted that economic growth rate (GDP growth rate) equates the growth rate of Y.

In this case, the labor share relative to the aggregate GDP (Υ) is given by:

$$S_L := \frac{wL}{\Upsilon} = \frac{1-\alpha}{1 + \alpha^2\left(\frac{1}{u} - 1\right)}$$

Since $\partial s_L / \partial u > 0$, the following result is obtained. The intuition behind this decline in s_L is straightforward: while the labor payment wL remains constant for a given Y, the aggregate GDP Υ increases as R&D activities intensify (i.e., as u decreases).

Result 2 *The labor share measured relative to aggregate GDP, S_L, necessarily declines as the allocation of resources to R&D increases (i.e., as u decreases), even though the labor share relative to final goods production, s_L, remains constant.*

Discussion 2 By assuming the input of capital goods into R&D and precisely defining GDP as Υ, we demonstrate that the decline in the labor share—a phenomenon increasingly discussed as a emerging stylized fact—can be generated even within the framework of a Cobb-Douglas Romer model. Specifically, our model provides a structural explanation for how the upsurge in R&D expenditures, as documented by Jones (2015) and OECD (2022), directly contributes to the diminishing labor share. While the Cobb-Douglas specification is often avoided for its rigid factor shares, we show that a rigorous accounting of R&D value-added allows this classic structure to bridge the gap between rising R&D efforts and the shifting functional distribution of income.

We next consider the asset share, defined as the sum of rental income from capital and monopoly profits accruing to firm owners:

$$S_A := \frac{rK + \Pi}{\Upsilon} = \frac{\alpha\left[1 + \alpha\left(\frac{1}{u} - 1\right)\right]}{1 + \alpha^2\left(\frac{1}{u} - 1\right)}.$$

By construction, the aggregate income shares satisfy $S_A + S_L = 1$.

The expression for S_A reveals a mechanism symmetric to that governing the labor share. As resources are reallocated toward R&D (a decline in u), the expansion of GDP through R&D-generated value added increases the income accruing to capital owners via both rental payments and monopoly profits. Since labor compensation wL remains tied to final-goods production, the additional value created by innovation is disproportionately captured by capital income. Consequently, S_A necessarily increases as R&D intensity rises (i.e., as u declines), implying $\partial S_A/\partial u < 0$. This finding highlights a distinct driving force behind the decline in the labor share.

In contrast to explanations based on rising markups, our mechanism preserves the constant markup implied by the Romer (1990) structure. Although monopoly profits are generated in equilibrium, their role is purely distributive: the decline in the labor share reflects the endogenous expansion of GDP through R&D activity, not an increase in market power.

While the Cobb-Douglas specification is frequently sidelined in contemporary studies of the labor share due to its structural rigidity (i.e., constant factor shares), our analysis demonstrates that such a framework is, in fact, capable of explaining the empirical trends documented by Jones (2015) and

OECD (2022). By reconciling the definition of GDP with the actual value-added of R&D, we show that the declining labor share–often attributed to more complex technological shifts—can emerge naturally even within the fundamental structure of the Romer model.

5 Extension: Human Capital

The baseline model treats capital as the sole input into R&D. We now extend the framework by introducing human capital explicitly into the innovation technology.

Let H denote human capital. Both physical and human capital are accumulated through final-good investment, so the resource constraint becomes

$$Y = \dot{K} + \dot{H} + C.$$

We modify the R&D technology as

$$\dot{A}(t) = \delta \frac{A(t) K_A(t)^{1-\psi} H(t)^{\psi}}{\Omega(t)}, \quad 0 < \psi < 1. \tag{22}$$

When $\psi \to 0$, the model converges to the baseline case.

Input composition Profit maximization yields

$$H = \frac{\psi}{1-\psi} K_A,$$

so the input ratio

$$\xi \equiv \frac{H}{K_A} = \frac{\psi}{1-\psi}$$

is constant in equilibrium.

Substituting into (22) gives

$$g_A = \delta(1-u)\xi^{\psi}. \tag{23}$$

Thus, the extension preserves the linear dependence of innovation growth on the R&D allocation $(1-u)$. Human capital modifies only the effective productivity parameter through the constant factor ξ^{ψ}.

To maintain a balanced-growth equilibrium, we continue to assume $\Omega = K$.

Free entry Under positive R&D,

$$\delta \frac{\tilde{V}}{\Omega} \xi^{\psi} = \frac{r}{1-\psi}.$$

Using $\varphi \equiv \psi^{\psi}(1-\psi)^{1-\psi}$, this condition can be written compactly as

$$\delta \tilde{v} \varphi = r.$$

This expression is structurally identical to the baseline condition, differing only by a constant productivity adjustment.

Balanced-growth equilibrium in the human-capital extension

Replacing the baseline free-entry condition (15) with the corresponding condition in the extension, (**??**), the BGP counterparts of (**??**) and (**??**) become

$$r^* = \theta \left[\delta(1-u^*)\xi^{\psi} + \frac{\alpha}{1-\alpha}\gamma \right] + \rho + n,$$
$$r^* = n + \frac{\alpha}{1-\alpha}\gamma + \frac{1-\alpha}{\alpha}\delta \, \varphi \, u^*,$$

where $\xi \equiv H/K_A = \psi/(1-\psi)$ and $\varphi \equiv \xi^{\psi}/(1+\xi) = \psi^{\psi}(1-\psi)^{1-\psi}$.

Eliminating r^* yields the equilibrium allocation:

$$\theta \left[\delta(1-u^*)\xi^{\psi} + \frac{\alpha}{1-\alpha}\gamma \right] + \rho = \frac{1-\alpha}{\alpha}\delta \, \varphi \, u^* + \frac{\alpha}{1-\alpha}\gamma.$$

Solving for u^* gives

$$u^* = \frac{\theta\delta\xi^{\psi} - (1-\theta)\frac{\alpha}{1-\alpha}\gamma + \rho}{\delta\xi^{\psi}\left[\frac{(1-\alpha)(1-\psi)}{\alpha} + \theta\right]},$$
$$1-u^* = \frac{\frac{(1-\alpha)(1-\psi)}{\alpha}\delta\xi^{\psi} + (1-\theta)\frac{\alpha}{1-\alpha}\gamma - \rho}{\delta\xi^{\psi}\left[\frac{(1-\alpha)(1-\psi)}{\alpha} + \theta\right]}.$$

Substituting $1-u^*$ into $g_A = \delta(1-u^*)\xi^{\psi}$ yields the long-run growth rate:

$$g^* = \frac{\frac{(1-\alpha)(1-\psi)}{\alpha}\delta\xi^{\psi} - \rho}{\frac{(1-\alpha)(1-\psi)}{\alpha} + \theta} + \frac{\frac{1-(1-\alpha)\psi}{1-\alpha}\gamma}{\frac{(1-\alpha)(1-\psi)}{\alpha} + \theta} + n.$$

The additive (two-channel) structure of long-run growth is preserved in this extension, with human capital affecting the innovation component through the constant term ξ^{ψ} (and hence φ) without altering the core allocation mechanism.

The main concern of this section is the implications of human capital distribution for income allocation. The income shares of human capital S_H and aggregate income $\Upsilon \equiv rK + rH + wL + \Pi$ are calculated as follows:

$$S_H = \frac{\alpha^2 \frac{\psi}{1-\psi}\left(\frac{1}{u}-1\right)Y}{\Upsilon}, \quad \Upsilon = \left[1 + \frac{\alpha^2}{1-\psi}\left(\frac{1}{u}-1\right)\right]Y.$$

We define the income share ratio between human capital and labor, $\sigma(u)$, as follows:

$$\sigma(u) \equiv \frac{S_H}{S_L} = \frac{\alpha^2\left(\frac{1}{u}-1\right)\xi}{1-\alpha}.$$

This immediately shows that $d\sigma(u)/du < 0$, implying that an increase in the R&D expenditure rate ($u \downarrow$) raises the income share of human capital relative to labor.

While the skill premium remains constant due to the rental nature of human capital and a fixed interest rate, the expansion of R&D activity reallocates economic resources toward skilled (cognitive) labor, thereby increasing its income share relative to unskilled labor.

Accordingly, we have the following result:

Result 3 *A reallocation of resources toward R&D (a decline in u) increases the income share of human capital relative to raw labor through reallocation effects.*

We next examine how R&D activity affects the functional distribution of labor income. In this version, human capital income is interpreted as part of labor compensation. Accordingly, we can define the (broad) labor distoribution ($S_{\tilde{L}}$) as follows:

$$S_{\tilde{L}}(u) = \frac{wL + rH}{\Upsilon} = \frac{1-\alpha+\frac{\psi\alpha^2}{1-\psi}\left(\frac{1}{u}-1\right)}{1+\frac{\alpha^2}{1-\psi}\left(\frac{1}{u}-1\right)}.$$

Differentiating $S_{\tilde{L}}(u)$ with respect to u yields

$$\frac{dS_{\tilde{L}}(u)}{du} = \frac{\alpha^2}{(1-\psi)u^2}\frac{(1-\alpha-\psi)}{\left[1+\frac{\alpha^2}{1-\psi}\left(\frac{1}{u}-1\right)\right]^2}.$$

which implies

$$\frac{dS_{\tilde{L}}(u)}{du}\left\{\begin{array}{c}>\\<\end{array}\right\}0 \Longleftrightarrow \psi\left\{\begin{array}{c}<\\>\end{array}\right\}1-\alpha.$$

Thus, the sign of $\frac{dS_{\tilde{L}}(u)}{du}$ is determined solely by $1-\alpha-\psi$. See Appendix for the discussion.

6 Conclusion

This paper revisits the Romer (1990) growth model to study the interaction between research and development (R&D), capital goods prices, and the functional distribution of income. Motivated by the simultaneous rise in R&D expenditure, the slowdown in economic growth, and the decline in the labor share observed in advanced economies, we develop a parsimonious framework that incorporates investment-specific technological change and capital-intensive R&D within a tractable growth model.

The first main result shows that a slowdown in capital-embodied technological progress reallocates resources toward R&D while reducing long-run economic growth. This mechanism provides a structural explanation for the coexistence of rising R&D intensity and secular stagnation—often referred to as the innovation paradox. Importantly, this result emerges without changes in market structure or markups, highlighting the role of capital goods prices and intertemporal allocation.

The second main result concerns the functional distribution of income. By explicitly capitalizing R&D activities in the measurement of aggregate GDP, we show that a declining labor share can arise endogenously even within a Cobb-Douglas production structure. Although factor shares relative to final-goods production remain constant, the expansion of GDP through R&D-generated value added mechanically reduces the labor share measured at the aggregate level. This finding reconciles classical production technologies with recent empirical evidence on declining labor shares.

We further extend the baseline model to incorporate human capital as an explicit input into R&D. While the core growth mechanism remains unchanged, this extension demonstrates that the decline in the broad labor share becomes conditional on the relative importance of cognitive inputs in innovation. At the same time, the model predicts an increase in the income share of human capital relative to raw labor as R&D intensity rises, offering a novel perspective on the distributional consequences of innovation.

The analysis deliberately focuses on steady-state equilibria in order to isolate the long-run accounting and price-based mechanisms emphasized in this paper. By abstracting from transitional dynamics, the model highlights properties that hold in the long run, independent of short-run adjustment paths. An important avenue for future research is to study how nonlinear R&D technologies and transitional dynamics shape the joint evolution of growth and income distribution. In addition, interpreting human capital as a skill component embedded in labor compensation, rather than as a rented input, would allow the framework to address changes in skill premia and wage inequality more directly.

A Labor Share with Human Capital

In Section 4, we obtain the following condition:

$$\frac{dS_{\tilde{L}}(u)}{du} \left\{ \begin{array}{c} > \\ < \end{array} \right\} 0 \Longleftrightarrow \psi \left\{ \begin{array}{c} < \\ > \end{array} \right\} 1 - \alpha.$$

Result A *If $\psi < 1 - \alpha$, an increase in the R&D expenditure share (i.e., a decrease in u) leads to a decline in the broad labor income share.*

Discussion A In the baseline model, the labor share always decreases as u decreases, but in the model with human capital accumulation, this holds conditionally. While not intended as a quantitative test, the condition for this to hold is $\psi < 1 - \alpha$, where ψ is the production contribution share of human capital in R&D activities, and $1 - \alpha$ is the production contribution share of labor in the production of final goods.

While our model is not intended to provide a direct explanation of recent labor market developments, the condition $\psi < 1 - \alpha$ appears broadly consistent with discussions of occupational wage changes in advanced economies. In particular, recent studies document heterogeneous wage dynamics across occupations, including relative wage growth in frontline and non-college jobs alongside weaker demand for certain knowledge-intensive tasks (Autor, 2019; Goos et al., 2014). Acemoglu and Restrepo (2020) further emphasize that automation technologies generate both displacement and productivity effects, with the net impact on labor demand depending on task reallocation.

We interpret these findings as suggestive background rather than direct empirical validation of our mechanism.

References

[1] Acemoglu, D., and R. Pascual (2020). Robots and jobs: Evidence from US labor markets. *Journal of Political Economy* 128(6), 2188–2244.

[2] Acemoglu, D. (2024). 2023 Klein Lecture—Capital and wages. *International Economic Review*, 13, 3-24.

[3] Aghion, P., Antonin, C., and Bunel, S. (2021). The innovation paradox: Why is productivity growth slowing down despite faster technological progress? *Economie et Statistique / Economics and Statistics*. No. 526-527, 13-25.

[4] Akcigit, Ufuk (2024). The innovation paradox. *International Monetary Fund, Finance & Development Magazine*, 61(3), 32-35.

[5] Antràs, Pol (2004). Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *B.E. Journal of Macroeconomics*, 4(1), 1-36.

[6] Autor, D. H., (2019). Work of the Past, Work of the Future. *AEA Papers and Proceedings*, 109, 1-32.

[7] Autor, D., Dorn, D., Katz, L. F., Patterson, C., and Van Reenen, J. (2020). The Fall of the Labor Share and the Rise of Superstar Firms. *Quarterly Journal of Economics*. 135(2), 645-709.

[8] Autor, D. H., F. Levy and R. J. Murnane (2003). The Skill Content of Recent Technological Change: An Empirical Exploration. *The Quarterly Journal of Economics*, 118(4), 1279-1333.

[9] Barro, R. J., and Sala-i-Martin, X. (2004). *Economic Growth* (2nd ed.). MIT Press.

[10] Blanchard, O., (2011). 2011 in review: Four hard truths. *iMFdirect*. https://blog-imfdirect.imf.org/2011/12/21/2011-in-review-four-hard-truths/ (accessed 21 December 2011).

[11] Byrne, D. M., and Pinto, E. P. (2015). The Recent Slowdown in High-Tech Equipment Price Declines and Some Implications for Business Investment and Labor Productivity. *FEDS Notes*. Washington: Board of Governors of the Federal Reserve System, March 26, 2015.

[12] Byrne, D. M., Fernald, J. G., and Reinsdorf, M. B. (2016). Does the United States have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity*, 2016(1), 109-182.

[13] Chirinko, R. S. (2008). The Elasticity of Substitution between Capital and Labor. *Journal of Macroeconomics*, 30(2), 645-658.

[14] Goos, Maarten. and Alan Manning (2007). Lousy and Lovely Jobs: The Rising Polarization of Work in Britain. *The Review of Economics and Statistics*, 89(1), 118-133.

[15] Greenwood, J., Hercowitz, Z., and Krusell, P. (1997). Long-run implications of investment-specific technological change. *The American Economic Review*, 87(3), 342-362.

[16] Grossman, G.M., E. Helpman, E. Oberfield and T. Sampson (2017). The Productivity Slowdown and the Declining Labor Share: A Neoclassical Exploration. *NBER Working Papers* 23853, National Bureau of Economic Research, Inc.

[17] Hall, R. E. (1988). Intertemporal Substitution in Consumption. *Journal of Political Economy*, 96(2), 339-357.

[18] Jones, C. I. (2015). The Facts of Economic Growth. *NBER Working Paper* No.21142.

[19] Jorgenson, D. W. (2001). Information technology and the US economy. *The American Economic Review*, 91(1), 1-32.

[20] Karabarbounis, L., and Neiman, B. (2014). The Global Decline of the Labor Share. *The Quarterly Journal of Economics*, 129(1), 61-103.

[21] Karabarbounis, L., and Neiman, B. (2018). Accounting for factorless income. *Brookings Papers on Economic Activity*, 2018(1), 167-228.

[22] Kuwahara, S. (2013). Dynamical analysis of the R&D-based growth model with a regime switch. *Journal of Economics (JEZN)* 108, 35-57.

[23] Kuwahara, S. (2019). Multiplicity and stagnation under the Romer model with increasing returns of R&D. *Economic Modelling*, 79, 86-97.

[24] Lovenheim, M. F., and Turner, S. E. (2017). The economic returns to for-profit colleges. *Brookings Papers on Economic Activity*, 2017(1), 1-68.

[25] Michaels, G., Natraj, A., & Van Reenen, J. (2014). Has ICT polarized skill demand? Evidence from eleven countries over twenty-five years. *Review of Economics and Statistics*, 96(1), 60-77.

[26] OECD (2022). *Main Science and Technology Indicators*, Vol. 2022(2).

`https://www.oecd.org/content/dam/oecd/en/publications/reports/2023/06/main-science-and-technology-indicators-volume-2022-issue-2_542daf66/1cdcb031-en.pdf`

[27] Rivera-Batiz, L. A., and P. M. Romer. (1991). Economic integration and endogenous growth. *The Quarterly Journal of Economics*, 106(2), 531-550.

[28] Romer, P. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5), Part 2, S71-S102.

[29] Schwellnus, C., M.Pak, P. Pionnier and E. Crivellaro (2018). Labour share developments over the past two decades: The role of technological progress, globalisation and " winner-takes most " dynamics. *OECD Economics Department* Working Papers, No. 1503.

[30] Segerstrom, P.S. (1998). Endogenous growth without scale effects. *American Economic Review*, 88, 1290-1310.

[31] Summers, L.H. (2014). U.S. economic prospects: Secular stagnation, hysteresis, and the zero lower bound. *Business Economics*, 49(2), 65–73.

[32] United Nations (2009). System of National Accounts 2008. European Commission, International Monetary Fund, Organisation for Economic Co-operation and Development, United Nations and World Bank. New York.

[33] Vissing-Jørgenson,A.,and Attanasio,O.P. (2002). Limited Asset Market Participation and the Intertemporal Elasticity of Substitution. *Journal of Political Economy*, 110(4), 825-853.

[34] Xu, S. (2017). Volatility risk and economic welfare. *Journal of Economic Dynamics & Control*, 80, 17-30.