A simple Schumpeterian growth model

Shiro Kuwahara

February 21, 2026

1 Introduction

This lecture introduces the model of Aghion and Howitt (1992), in which long-run growth arises from quality improvements in intermediate goods.

Conceptually, quality improvement and the variety expansion mechanism developed by Romer (1990) are closely related approaches to technological progress. In both frameworks, growth is driven by monopoly rents associated with successful innovation.

Aghion and Howitt (1992), however, emphasized a distinctive feature of their framework: innovation takes the form of creative destruction, whereby new entrants displace incumbent monopolists. For this reason, they explicitly framed their approach as a " Schumpeterian " growth model. While the underlying economic forces share important similarities with the variety-expansion model, the Schumpeterian framework highlights firm turnover and the dynamic replacement of rents more explicitly.

Grossman and Helpman (1991) provided an early and influential synthesis, developing both variety-expansion and quality-ladder models within a unified general equilibrium framework and linking innovation to international trade. Although quality-ladder models were developed contemporaneously by Grossman and Helpman (1991) and Aghion and Howitt (1992), the latter 's Schumpeterian terminology subsequently became standard in the literature.

Over time, the Schumpeterian quality-ladder approach has become a dominant paradigm in the modern growth literature. One reason is that it naturally incorporates creative destruction, firm turnover, and innovation races—features that align closely with empirical evidence on firm dynamics and productivity growth. Moreover, the framework connects growth theory to industrial organization, competition policy, and patent protection in a tractable way.[1]

he Schumpeterian framework has also provided the foundation for a large quantitative and empirical literature. Klette and Kortum (2004) and Lentz and Mortensen (2008) integrate quality-ladder models with firm dynamics, allowing innovation, entry, and growth to be studied in a unified framework. Aghion et al. (2005) highlight the " escape competition " effect, linking market structure to innovation incentives. More recently, Akcigit and Kerr (2018) incorporate innovation networks and knowledge spillovers into Schumpeterian growth models using firm-level data.

Together, these contributions illustrate how the creative-destruction paradigm has evolved into a central framework for analyzing innovation, competition, and long-run growth in modern macroeconomics.

While alternative mechanisms of innovation remain important, much of the contemporary empirical literature adopts a Schumpeterian structure as its organizing framework.

This paper is presented as follows. Section 2 develops the model. Section 3 presents the derivation of the steady state of the model and explains the properties obtained from the model. In section 4, capital accumulation is introduced. Section 5 concludes the paper.

2 Basic Model

The model of this study consists of a household with the spirit of capitalism and a production sector with endogenous technological progress. The time is continuous, and the final goods are used as a numeraire.

2.1 Schumpeter Economy

The Schumpeter economy is the one where innovation is undertaken. This study adopts an Aghion and Howitt (1992)-type R&D structure, and follows the Romer (1990) production structure in that final goods are produced using a continuum of intermediate goods. The economy consists of three sectors: final goods production, intermediate goods production, and R&D.

[1]See Aghion and Howitt (1998) and Aghion, Akcigit, and Howitt (2014) for comprehensive treatments of the Schumpeterian growth framework.

Labor is the only primary factor of production and is supplied inelastically by the population. It is allocated between final goods production and R&D activities:

$$L = L_Y + L_A.$$

Final output is used either for consumption or for the production of intermediate goods:

$$Y = C + Z.$$

Each intermediate good is indexed by $i \in (0, N)$. N is fixed and sufficiently large. Each sector features a vertical quality ladder along which innovations occur.

In sector i, quality levels are indexed by $m_i = 1, 2, \cdots, M_i$, with corresponding quality levels $\lambda, \lambda^2, .., \lambda^{M_i}$, where $\lambda > 1$ denotes the step size of innovation.

The current frontier quality in sector i is $q(i) \equiv \lambda M_i$. Since quality-adjusted intermediate goods within a sector are perfect substitutes, only the highest-quality good is produced in equilibrium.

We assume constant returns to scale and no complementarity across intermediate goods. Letting $x(i)$ denote demand for the highest-quality intermediate good in sector i, final output is given by

$$Y = L_Y^{1-\alpha} \int_0^N \{q(i)x(i)\}^\alpha di, \quad 0 < \alpha < 1, \tag{1}$$

where L_Y is the labor supply allocated to final goods production.

The first order condition (FOC) of production is obtained as

$$\frac{\partial Y}{\partial x(i)} = p(i), \quad \text{and} \quad \frac{\partial Y}{\partial L_Y} = w_Y, \tag{2}$$

where $p(i)$ and w_Y are the price of the ith intermediate goods of the highest quality and wage rate offered in the final goods production sector, respectively.

R&D firms facilitate technological progress; they create a design that is one grade higher in terms of quality than a incumbent highest quality design. The R&D activities of the firms take place at the beginning of each period, and the results are immediately evident. A successful research firm retains exclusive rights for the new higher quality intermediate goods. This exclusive right is referred to as a "patent."

In this study, one unit of intermediate goods is produced by η units of final goods. Hence, the firm that produces the ith intermediate good maximizes the profit such that

$$\pi(i) = p(i)x(i) - \eta\, x(i). \tag{3}$$

The monopolist firm that holds the patent for the current high quality good maximizes its profit by considering price as a control variable. Therefore, the FOC of the monopoly firm in the ith sector with quality M_i yields the following:

$$x(i) = \left[\frac{\alpha^2}{\eta}\right]^{\frac{1}{1-\alpha}} L_Y q(i)^{\frac{\alpha}{1-\alpha}}, \qquad \text{and} \quad p(i) = \frac{\eta}{\alpha}. \tag{4}$$

With regard to monopolistic pricing, we have the following three conditions. First, each unit of the highest quality good is equivalent to $\lambda(>1)$ units of the good with the next best quality. Second, a good that is one grade lower than the highest quality good is supplied at marginal cost η because the patent for this quality grade expires. Third, the different quality grades are perfect substitutes if they are weighted by the quality level. Based on the above conditions, it follows that $p(i) < \lambda\eta$ is necessary for the firm to create the highest quality good to monopolize the demand for that good. Therefore, a combination of (4) and $p(i) < \lambda\eta$ indicates that (4) is the optimal condition under the assumption that $1/\alpha < \lambda$. The following discussion is developed such that it satisfies the present assumption.[2] Thus, only the highest quality goods are supplied.

The aggregate index of quality is defined as

$$Q \equiv \int_0^N q(i)^{\frac{\alpha}{1-\alpha}} di. \tag{5}$$

Substituting (4) into (1) and using (5) to rearrange the same, we obtain the aggregate output Y as

$$Y = \alpha^{\frac{2\alpha}{1-\alpha}} \eta^{-\frac{\alpha}{1-\alpha}} L_Y Q. \tag{6}$$

Thus, the final goods production growth rate is the linear addition of the growth rates of labor supply and quality index. We also note the intermediate goods input of final goods, which is denoted by X and obtained by

[2]This assumption implies that the width of one innovation is sufficiently large. If $1/\alpha < \lambda$, the optimal pricing is given as $p(i) = \lambda\eta$. This pricing does not alter the main framework of the model; therefore, we assume $1/\alpha < \lambda$ throughout this study.

aggregating (4) across sectors and using

$$Z = \eta \int_0^N x(i)di = \alpha^{\frac{2}{1-\alpha}} \eta^{-\frac{\alpha}{1-\alpha}} L_Y Q (= \alpha^2 Y). \tag{7}$$

Substituting (7) and (6) into the final goods resource constraint equation $Y = C + Z$, we obtain the aggregate consumption as

$$C = (1-\alpha^2)Y = (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}} \eta^{-\frac{\alpha}{1-\alpha}} L_Y\, Q. \tag{8}$$

Thus, the allocation between consumption C and intermediate input Z is fixed in this basic model. We do not make this assumption in the latter part of this study.

By using (3) and (4), the profit of the monopoly firm of the ith sector with a patent for goods of quality M_i is obtained as follows:

$$\begin{aligned} \pi(i) &= \left(\frac{1}{\alpha} - 1\right) \eta x(i) \\ &= (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} L_Y q(i)^{\frac{\alpha}{1-\alpha}} \\ &= \left(\frac{1}{\alpha} - 1\right) \frac{Y}{Q} q(i)^{\frac{\alpha}{1-\alpha}}. \end{aligned} \tag{9}$$

2.2 R&D Activities

The specification adopted here follows this line by making R&D success depend both on relative R&D effort and on relative quality position, thereby combining frontier-based innovation incentives with a difficulty-based mechanism that ensures scale-neutral long-run growth.

It is presumed that R&D activities for the purpose of innovating higher quality levels are conducted using labor, and the success of R&D stochastically depends on the labor input. With regard to the behavior of the R&D firm and aggregate quality index, we basically follow Barro and Sala-i-Martin (1995 Ch7).

When innovation occurs in a sector, the probability that firm j in that sector will be granted a patent is assumed to be proportional to its share of the R&D input in the ith sector (that is, $L_A(i)^j/L_A(i)$, where $L_A(i)^j$ and $L_A(i)$ represent the R&D input for the ith sector by firm j and the aggregate R&D input for the ith sector, respectively).

Therefore, from the above assumptions, it is shown that the profit for sector i of R&D firm j is

$$\max_{L_A(i)^j} \quad \frac{\mu(i)L_A(i)^j}{L_A(i)} v(i) - w_A L_A(i)^j.$$

where w_A is the wage rate offered in the R&D sector and $v(i)$ is the ith goods innovation's expected current value of profit. The presence or absence of investment in R&D activities is determined as follows. If $\frac{\mu L_A(i)^j}{L_A(i)} v(i) < w_A L_A(i)^j$ holds, R&D is not profitable. Consequently, the R&D input stops and equilibrium is attained without R&D; therefore the probability of R&D success is 0, that is, $L_A(i) = 0$ and $\mu(i) = 0$. In the case of no R&D, the quality of the intermediate goods would remain constant over time. If $\frac{\mu L_A(i)^j}{L_A(i)} v(i) = w_A L_A(i)^j$, a positive amount of labor would be devoted to R&D activities and the market would be in equilibrium. Furthermore, no arbitrage on labor market leads to $w \equiv \max\{w_Y, w_A\}$. Therefore, in equilibrium, $w = w_Y = w_A$ must hold if R&D takes place, and $w = w_Y > w_A$ must hold if there is no R&D. The above points can be summarized as follows:

$$\mu(i)v(i) = wL_A(i), \quad \text{for} \quad L_A(i) > 0. \tag{10}$$

Thus, if R&D has a zero probability, it holds with $L_A(i) = 0$ and $\mu(i) = 0$. The case of (10) presents endogenous growth with a positive economic growth rate, and the case with $L_A(i) = 0$ and $\mu(i) = 0$ does exogenous growth with no-growth or poverty traps.

Substituting w in (2) into the equation in (10) yields

$$v(i) = \frac{1-\alpha}{\mu(i)} \frac{Y}{L_Y} L_Y(i) \tag{11}$$

First, we assume that the former case depicts a steady state with positive long-run growth. Under this assumption, the time differentiation of (10) is calculated as

$$rv(i) = \pi(i) + \dot{v}(i) - \mu(i^+)v(i), \quad \text{or equivalently}$$
$$r + \mu(i^+) = \frac{\pi(i)}{v(i)} + g_{v(i)}, \tag{12}$$

where $\mu(i^+)$ denotes the R&D success probability of this sector with one step higher quality.

Substituting (9) and (11) into (12), we obtain

$$r + \mu(i^{+}) = \frac{\mu(i)\left(\frac{1}{\alpha} - 1\right)\frac{Y}{Q}q(i)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)\frac{Y}{L_Y}L_A(i)} + \frac{\dot{v(i)}}{v(i)}. \tag{13}$$

We assume that the ith sector's innovation success is determined by two relative factors: relative R&D labor input share $L_A(i)/L$ and ith sector's relative quality rank $q(i)/Q$. The former has a positive effect and the latter negative. We further assume that these two effects are linearly related on the Poisson arrival rate of innovation success. Thus, it is specified as follows:[3]

$$\mu(i) = \xi \frac{QL_A(i)}{q(i)^{\frac{\alpha}{1-\alpha}}L}, \qquad \xi > 0. \tag{14}$$

We assume symmetric equilibrium across intermediate goods sectors. Using $\frac{\dot{v}(i)}{v(i)} = n$ in a steady state, which can be obtained from (9) and (14), (13) in a steady state is produced as

$$\mu = \frac{\xi}{\alpha}l + n - r, \tag{15}$$

where $l \equiv L_Y/L$ is the rate of labor division on final goods production.

(14) and (15) yields

$$L_A(i) = \frac{q(i)^{\frac{\alpha}{1-\alpha}}L}{\xi Q}\left[\frac{\xi}{\alpha}l + n - r\right]. \tag{16}$$

Aggregating (16) about i yields the aggregate R&D spending, denoted by L_A, as

$$L_A = (1-l)L = \int_0^N L_A(i)di = L\left[\frac{1}{\alpha}l - \frac{r-n}{\xi}\right]. \tag{17}$$

[3] L and Q are respectively the quantitative and qualitative indices as shown by the final goods production in (6), where the linear relationship between population and quality, and GDP can be obtained. Therefore, (14) implies that R&D success depends on positive aggregate quality level and private R&D input, and a negative economic quantitative scale and private quality level. Modern Schumpeterian growth models typically eliminate scale effects through difficulty-based specifications, in which innovation becomes progressively harder as sectors approach the technological frontier (see, e.g., Segerstrom, 1998). At the same time, recent contributions emphasize step-by-step innovation and the role of relative distance to the frontier in shaping innovation incentives (Aghion et al., 2005). In our setting, the property that μ depends on $\frac{q(i)^{\frac{\alpha}{1-\alpha}}}{Q}$ follows Aghion et al., (2005), and the property that μ depends on $\frac{L_A(i)}{L}$ stems from yje concept proposed by Segerstrom (1988).

Hence, L_A is proportional to L for a given variable r. (17) immediately yields the relationship between r and l as follows:

$$r - n = \left\{ \left(\frac{1}{\alpha} + 1 \right) l - 1 \right\} \xi, \quad \text{or} \quad l = \frac{1}{1+\alpha} \left(\frac{r-n}{\xi} + \alpha \right). \tag{18}$$

Uniting (15) and (18), we obtain the following equilibrium μ as a function of l;

$$\mu = \xi(1 - l). \tag{19}$$

Thus, the innovation probability of the economy demonstrates a linearity relationship against the rate of labor input on R&D activity.

3 Long-run Growth and its Properties

3.1 Conditions of Steady state by production

From the dynamics of Q defined in (5), the increment of the ith sector's innovation $R(i) \equiv q(i^+)^{\frac{\alpha}{1-\alpha}} - q(i)^{\frac{\alpha}{1-\alpha}}$ is calculated as $R(i) = q(i)^{\frac{\alpha}{1-\alpha}}(\lambda^{\frac{\alpha}{1-\alpha}} - 1)$. Therefore, the aggregate dynamics of Q are

$$E(\dot{Q}) = \int_0^N R(i)di = \mu(\lambda^{\frac{\alpha}{1-\alpha}} - 1)Q. \tag{20}$$

From (19) and (20), the dynamics of Q as a function of l are derived as follows:

$$g_Q = \mu(\lambda^{\frac{\alpha}{1-\alpha}} - 1) = \xi(1 - l)\Lambda, \tag{21}$$

where $\Lambda \equiv \lambda^{\frac{\alpha}{1-\alpha}} - 1 > 0$ and $g_Z \equiv \dot{Z}/Z$. Since $\partial\Lambda/\partial\lambda > 0$, Λ is the parameter that immediately captures the scale of one innovation.

From (10) and (14), the aggregate market value of R&D firms V is calculated as

$$V = \int_0^N v(i)di = \int_0^N \frac{wL_A(i)}{\mu(i)}di = \frac{(1-\alpha)Y}{\xi l}, \tag{22}$$

where we use $w = (1-\alpha)Y/(lL)$ for this derivation. Since we assume symmetric equilibrium for the household, and only the asset of this economy is the equity of R&D firms, the per capita asset holding A is denoted as

$$v = \frac{V}{L} = \frac{(1-\alpha)y}{\xi l}. \tag{23}$$

Substitution of y from (6) into (23) yields

$$v = \frac{1-\alpha}{\xi}\alpha^{\frac{2\alpha}{1-\alpha}}\eta^{-\frac{1}{1-\alpha}}Q. \tag{24}$$

This equation implies

$$g_v = g_Q = \xi(1-l)\Lambda. \tag{25}$$

3.2 Household

To close the model, we need the household. We assume a normally-utilized dynasty household with the utility

$$U = \int_0^\infty u(c)\, e^{-\rho t} dt, \tag{26}$$

where ρ and $u(c)$ are the subjective discount rate, and an instantaneous utility function of the representative household, respectively. The representative household has the following budget constraint:

$$\dot{a} = ra + w - c - na, \tag{27}$$

where a, r and w are per capita asset holdings, the interest rate and wage rate, respectively. n is a population growth rate that is assumed to be non-negative throughout the study.

We specify the instantaneous utility function as follows

$$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \tag{28}$$

Under this specification, the optimizing condition for the consumption growth rate is derived as

$$\sigma\frac{\dot{c}}{c} = r - n. \tag{29}$$

3.3 Derivatoin of Steady states

This equation implies that asset holding in the economy is proportionally related to the technological level, and it grows at the same rate as the quality index. (8) implies that the per capita consumption $c \equiv C/L$ grows at the same rate as the quality index. Thus, the steady state, wherein all variables grow at constant rates and the Euler equation (29) is satisfied, can exist.

Taking $C = cL$ into account, the time differential on (8) and (23) provides the following steady state growth rate:

$$g_c = g_Y - n = g_y = g_A = g_Q. \tag{30}$$

Substituting (8), (23) and (30) into (29), we obtain

$$\sigma g_Q = r - n - \rho. \tag{31}$$

Eliminating $r - n$ by using (18), (21), and (31), we can analytically obtain the equilibrium division of labor to production,

$$l^* = \frac{\frac{\rho}{\xi} + 1 + \sigma\Lambda}{\left(\frac{1}{\alpha} + 1\right)}, \tag{32}$$

From (21) and (**??**)Thus, we have the following Proposition:

Proposition 1 (Long-run growth) *The growth rate of the steady states is given as follows:*

$$g_y^* = g_Q^* = \underbrace{\frac{\frac{1}{\alpha} + \beta(1+\alpha) - \frac{\rho}{\xi}}{\left(\frac{1}{\alpha} + 1\right) + \beta(1+\alpha) + \sigma\Lambda}}_{\mu^*}\Lambda. \tag{33}$$

Proof) From (21), (30), and (32), we obtain (33) (Q.E.D.).

Equation (33) implies that a higher efficiency of R&D, ξ and Λ, and a lower subjective discount rate, ρ, accelerate the growth rate. These properties are basically shared with the usual R&D-based growth model.

Condition for long-run positive endogenous growth We have assumed the positive profitability of R&D, namely holding equality with (10). $l \in (0, 1)$ is necessary for the steady state obtained above to be a feasible equilibrium that is consistent with the positive R&D investment. It should be noted that (32) ensure $l^* > 0$, which implies labor allocation on final goods prodction can not be too large to run off the upper bound of l, namely $l = 1$.

Since (32) shows that $l^* > 0$ constantly holds, the restriction is eventually determined to be $l^* < 1$, which yields

$$\xi > \alpha\rho. \tag{34}$$

Namely, we have followsing Proposition:

Proposition II (No growth trap) *If an economy has low R&D efficiency, and is necessary for positive endogenous growth.*

Proof) For the positive growth, condition $l^* \in (0, 1)$ is necessary. $l^* > 0$ is trivial, and $l^* < 0$ is given in (34) (Q.E.D.).

This property is also standardly shared by the R&D-based growth model.

References

[1] Aghion, Philippe, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt (2005), " Competition and Innovation: An Inverted-U Relationship, "*Quarterly Journal of Economics*, Vol. 120, No. 2, pp. 701–728.

[2] Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323–351.

[3] Aghion, Philippe, and Peter W. Howitt (1998), *Endogenous Growth Theory*. MIT Press, Place: Cambridge, MA, USA

[4] Aghion, P., U. Akcigit, and P. Howitt (2014), " What Do We Learn from Schumpeterian Growth Theory? " in Handbook of Economic Growth, Vol. 2B, ed. by P. Aghion and S. N. Durlauf, pp. 515?563. Amsterdam: Elsevier (North-Holland).

[5] Grossman, G., Helpman, E., 1991. Innovation and Growth in the Global Economy. Cambridge: MIT Press.

[6] Lentz, Rasmus, and Dale T. Mortensen (2008), " An Empirical Model of Growth through Product Innovation, "*Econometrica*, Vol. 76, No. 6, pp. 1317?1373.

[7] Klette, Tor Jakob, and Samuel Kortum (2004), " Innovating Firms and Aggregate Innovation, "*Econometrica*, Vol. 72, No. 3, pp. 986–1018.

[8] Romer, P., 1990. Endogenous technological change. Journal of Political Economy 98(5), S71–S102

[9] Segerstrom, Paul S. (1998), "Endogenous Growth without Scale Effects," *American Economic Review*, Vol. 88, No. 5, pp. 1290?1310.

[10] Schumpeter, Joseph A. (1934), The Theory of Economic Development: An Inquiry into Profits, Capital, Credit, Interest, and the Business Cycle. Cambridge, MA: Harvard University Press. Originally published as Theorie der wirtschaftlichen Entwicklung: eine Untersuchung uber Unternehmergewinn, Kapital, Kredit, Zins und den Konjunkturzyklus. Leipzig: Duncker & Humblot, 1911.

Japanese translation: ヨーゼフ・A・シュンペーター（塩野谷祐一・中山伊知郎・東畑精一訳）（1977）『経済発展の理論』岩波文庫.