

Market Competitiveness, Structural Thresholds, and Regime Selection in Growth

Shiro Kuwahara*
University of Hyogo

February 20, 2026

Abstract

This paper develops a general-equilibrium growth model in which market competitiveness evolves endogenously as a state variable through the interaction of innovation and imitation. Competitiveness is defined as the stock share of competitive intermediate varieties and evolves only through entry and imitation; it is therefore not a jump variable under rational expectations.

The equilibrium dynamics reduce to a one-dimensional differential equation whose steady-state condition is characterized by a quadratic polynomial in competitiveness. For a non-empty region of the parameter space, two interior steady states arise: the lower root is locally stable, while the higher root constitutes an unstable structural threshold separating distinct basins of attraction.

Regime selection is thus governed by structural state dynamics rather than expectation-based coordination. The parameter space is partitioned by three regime boundaries that intersect at a unique point, organizing the coexistence of a competitive no-entry regime and a sustained-growth regime.

Keywords: Endogenous Growth; Market Structure; Imitation; Threshold Dynamics; Equilibrium Multiplicity; History Dependence.

JEL: O41; C62; D90; L16

*E-mail: kuwahara@em.u-hyogo.ac.jp

1 Introduction

How does market structure shape long-run growth? Standard endogenous growth models typically treat market competitiveness as exogenous or focus on balanced-growth equilibria without fully characterizing global dynamics. This paper develops a framework in which market competitiveness evolves endogenously as a state variable through the interaction of innovation and imitation.

Competitiveness is defined as the stock share of competitive intermediate varieties. Because this share is inherited from past entry and imitation, it is a predetermined state variable rather than a jump variable under rational expectations. In this framework, the equilibrium reduces to a one-dimensional dynamic system in competitiveness, permitting a complete characterization of steady states and global dynamics.

Market structure plays a central role in modern growth theory. In the Romer (1990) framework and its extensions, monopoly power provides incentives for innovation, while competitive forces discipline incumbents and enhance productive efficiency. In many endogenous growth models, however, the degree of competition is either treated as exogenous or adjusts instantaneously, so that long-run outcomes are characterized under a given market configuration.

Schumpeterian models introduce creative destruction and turnover of monopoly positions, allowing for dynamic replacement of incumbents. Yet even in these settings, aggregate market competitiveness does not evolve as an independent macroeconomic state variable governing the trajectory of the economy. While the competitive environment affects innovation incentives, it is not itself subject to gradual endogenous evolution at the aggregate level. What is missing is a framework in which market competitiveness evolves endogenously as a state variable and feeds back into innovation and growth dynamics in general equilibrium.

This paper develops such a framework. We construct a general-equilibrium model in which monopolized and competitive sectors coexist, and the share of competitive intermediate varieties evolves through the interaction of innovation and imitation. The resulting general-equilibrium feedback reduces to a one-dimensional nonlinear differential equation in the competitiveness state variable. This reduction permits a complete characterization of steady states, stability, and threshold-driven regime selection.

For a non-empty region of the parameter space, the reduced quadratic equation admits two interior roots. The lower root is locally stable, while the higher root constitutes an unstable structural threshold separating distinct basins of attraction. Long-run growth outcomes then depend on the initial

degree of competition: the economy converges either to a competitive no-entry regime or to a sustained-growth regime.

Importantly, competitiveness is a predetermined state variable governed by structural dynamics and is not freely adjustable by forward-looking agents. Although the higher root satisfies the static equilibrium conditions, it cannot be selected through expectations or coordination. Instead, it functions purely as a structural threshold that separates distinct dynamic trajectories.

Regime selection is therefore governed by structural state dynamics rather than by self-fulfilling beliefs. Small differences in initial market structure can generate persistent divergence in long-run growth regimes, even under perfect foresight and identical preferences. More broadly, the framework provides a tractable environment for analyzing how competition policy and innovation incentives jointly shape long-run growth regimes.

The paper makes three theoretical contributions.

First, it endogenizes aggregate market competitiveness as a macroeconomic state variable. Whereas many growth models take the competitive environment as given, the present framework allows market structure itself to evolve through the interaction of innovation and imitation. This generates a dynamic feedback between entry incentives and competitive intensity, making market structure an active determinant of long-run outcomes.

Second, the model admits a sharp analytical characterization. Despite nonlinear general-equilibrium interactions, the steady-state condition reduces to a quadratic equation in a single state variable. This dimensional reduction permits a complete classification of steady states and their stability properties in closed form.

Third, the paper delivers a belief-free mechanism of regime selection. Multiplicity arises from deterministic state dynamics rather than expectation-based coordination. Because the dynamics are one-dimensional, stability is fully characterized by the sign of the derivative at each root. When coexistence obtains, the unstable root acts as a structural threshold that partitions the state space into distinct basins of attraction, so that long-run outcomes depend solely on the initial degree of competition.

The paper relates to several strands of the literature. In Romer-type variety-expansion models, monopoly power is typically permanent or governed by exogenous patent expiration, while in Schumpeterian models innovation replaces incumbents through creative destruction. In both cases, however, aggregate market competitiveness does not evolve as an endogenous state variable governing regime selection.

A similar limitation appears in models featuring the coexistence of monopolized and competitive sectors (e.g., Matsuyama, 1999; Iwaisako and Futagami, 2003; Kuwahara, 2007). Although these frameworks incorporate

structural heterogeneity, the degree of aggregate competitiveness is not modeled as an independent state variable shaping long-run equilibrium outcomes.

The present analysis also connects to recent work on growth and market structure (e.g., Aghion et al., 2021; Peters, 2020). Whereas this literature studies how market structure influences growth incentives, the mechanism identified here isolates a deterministic channel through which structural nonlinearities generate threshold-driven regime transitions. Because the equilibrium dynamics reduce to a one-dimensional system, the source of multiplicity is transparent and structural rather than belief-driven.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the steady states and analyzes their stability. Section 4 concludes.

2 The Model

2.1 Production

We build on a Romer (1990) variety-expansion environment. The key modeling objective is to endogenize the endogenous evolution of aggregate market competitiveness, summarized by the share of competitive intermediate varieties. A variety structure allows monopolized and competitive varieties to coexist, which is essential for capturing gradual, history-dependent transitions in market structure. We abstract from within-sector CES substitution to keep within-regime markups simple and to obtain a low-dimensional characterization of the equilibrium dynamics.

We consider a continuous-time general equilibrium economy with final-goods production, intermediate-goods production, and entry (introduced below). Final goods are produced competitively using labor and a continuum of intermediate inputs, with the final good as numeraire:

$$Y = L_Y^{1-\alpha} \int_0^A \tilde{X}(i)^\alpha di, \quad \alpha \in (0, 1), \quad (1)$$

where Y is final output, L_Y labor in final production, A the measure of intermediate varieties, and $\tilde{X}(i)$ the quantity of intermediate input $i \in [0, A]$.

Perfect competition in the final-goods sector implies

$$\frac{\partial Y}{\partial L_Y} = (1 - \alpha) \frac{Y}{L_Y} = w, \quad \frac{\partial Y}{\partial \tilde{X}(i)} = \alpha L_Y^{1-\alpha} \tilde{X}(i)^{\alpha-1} = p(i), \quad (2)$$

where w is the real wage and $p(i)$ is the price of input i . Equation (2) implies

the conditional demand

$$\tilde{X}(i) = \left(\frac{\alpha}{p(i)} \right)^{\frac{1}{1-\alpha}} L_Y. \quad (3)$$

One unit of an intermediate good requires one unit of the final good as input, so marginal cost is one. Final output is used for consumption and intermediate production, so $Y = C + X$, where $X \equiv \int_0^A \tilde{X}(i) di$. Operating profits in sector i are

$$\tilde{\Pi}(i) = p(i)\tilde{X}(i) - \tilde{X}(i). \quad (4)$$

At each date, some varieties are supplied competitively and others by patent-holding monopolists. In the competitive sector ($i \in \mathcal{C}$), price equals marginal cost, $p_c = 1$, and (3) gives

$$i \in \mathcal{C} \Rightarrow \tilde{X}_c = \alpha^{\frac{1}{1-\alpha}} L_Y, \quad p_c = 1. \quad (5)$$

In the monopolized sector ($i \in \mathcal{M}$), the patent holder sets price to maximize (4) subject to (3), yielding

$$i \in \mathcal{M} \Rightarrow \tilde{X}_m = \alpha^{\frac{2}{1-\alpha}} L_Y, \quad p_m = \frac{1}{\alpha}. \quad (6)$$

Let $s(t) \in [0, 1]$ denote the share of intermediate varieties operating under competition at time t . Then $s(t)A(t)$ varieties are competitive and $(1 - s(t))A(t)$ are monopolized. Given symmetry within each regime, aggregate allocations depend on market structure only through $s(t)$.

Defining

$$\Phi(s; z) \equiv (1 - s) + \alpha^{-\frac{z}{1-\alpha}} s, \quad \text{for } z \in \{\alpha, 1\},$$

and using (5)–(6), aggregate intermediate demand is

$$X = \alpha^{\frac{2}{1-\alpha}} A \Phi(s; 1) L_Y. \quad (7)$$

Since Φ is linear and increasing in s , holding A and L_Y fixed, a higher s raises aggregate input efficiency and thereby aggregate output.

Substituting (5)–(6) into (1), aggregate final output is

$$Y = \alpha^{\frac{2\alpha}{1-\alpha}} A \Phi(s; \alpha) L_Y. \quad (8)$$

Finally, monopoly profit per monopolized variety is

$$\tilde{\Pi}_m = \left(\frac{1}{\alpha} - 1 \right) \tilde{X}_m = \alpha(1 - \alpha) \frac{Y}{A \Phi(s; \alpha)}. \quad (9)$$

2.2 Investment for Entry

Entry corresponds to the creation of a new intermediate variety. Let \dot{A} denote the flow of new designs. A successful entrant obtains a patent and earns monopoly profits until imitation occurs.

The value of a patent \tilde{V} satisfies

$$r\tilde{V} = \dot{\tilde{V}} + \tilde{\Pi}_m - \mu\tilde{V}, \quad (10)$$

where μ is the exogenous imitation rate. We treat μ as exogenous in order to isolate the general-equilibrium interaction between innovation and market structure. The qualitative results rely on the coexistence of innovation and imitation, rather than on the specific microfoundation of imitation.

New designs are produced using labor L_A according to

$$\dot{A} = \delta AL_A, \quad (11)$$

where $\delta > 0$ is entry efficiency.

Entry is competitive. Letting L_A^j denote labor employed by entrant j , profits are

$$\tilde{\Pi}_R = \frac{L_A^j}{L_A} \dot{A} \tilde{V} - wL_A^j.$$

Free entry implies the complementary-slackness conditions

$$\tilde{V}\delta A \leq w, \quad L_A \geq 0, \quad (\tilde{V}\delta A - w)L_A = 0. \quad (12)$$

Condition (12) determines the endogenous regime switch between growth and stagnation.

Using (11), the growth rate of knowledge is

$$g_A = \delta L_A.$$

Hence,

$$g_A > 0 \iff \tilde{V}\delta A = w, \quad g_A = 0 \iff \tilde{V}\delta A < w.$$

Combining (2), (8), and the entry condition yields

$$\tilde{V} = \frac{1-\alpha}{\delta} \alpha^{\frac{2\alpha}{1-\alpha}} \Phi(s; \alpha). \quad (13)$$

3 Dynamics and Steady States

The dynamic system consists of the joint evolution of market competitiveness and labor allocation. We first derive the law of motion for market competitiveness and then characterize the general-equilibrium dynamics.

3.1 Dynamics

We first characterize the law of motion for market competitiveness. We then derive the general-equilibrium dynamics of labor allocation.

3.1.1 Dynamics of market competitiveness.

Let $s(t) \in [0, 1]$ denote the share of competitive intermediate varieties and $\mathcal{M}(t) \equiv (1 - s(t))A(t)$ the measure of monopolized varieties. Differentiating $\mathcal{M} = (1 - s)A$ yields

$$\frac{\dot{\mathcal{M}}}{\mathcal{M}} = -\frac{\dot{s}}{1-s} + \frac{\dot{A}}{A}. \quad (14)$$

The measure of monopolized varieties increases through entry, $\dot{A} = \delta AL_A$, and decreases through imitation at rate $\mu(1 - s)A$. Using $L_A = (1 - u)L$, the law of motion for \mathcal{M} is

$$\dot{\mathcal{M}} = \delta A(1 - u)L - \mu(1 - s)A. \quad (15)$$

Eliminating \mathcal{M} from (14) and (15) gives the law of motion for market competitiveness:

$$\dot{s} = (1 - s)\mu - s\delta(1 - u)L. \quad (16)$$

Equation (16) makes clear that imitation increases s , whereas entry decreases s by expanding the set of monopolized varieties.

3.1.2 General-equilibrium dynamics.

Labor is allocated either to final-good production (L_Y) or to entry activities (L_A). Let $u \equiv L_Y/L$ so that $L_Y = uL$ and $L_A = (1 - u)L$. Using the production function (8) and the law of motion for knowledge (11), output growth satisfies

$$\frac{\dot{Y}}{Y} = \frac{\dot{u}}{u} + \delta(1 - u)L + \Gamma_Y(s)\dot{s}, \quad (17)$$

where the derivative of $\Phi(s; \alpha)$ with respect to time satisfies

$$\frac{\dot{\Phi}(s; \alpha)}{\Phi(s; \alpha)} = \Gamma_Y(s)\dot{s},$$

and $\Gamma_Y(s)$ is defined by

$$\Gamma_Y(s) \equiv \frac{\alpha^{-\frac{\alpha}{1-\alpha}} - 1}{\Phi(s; \alpha)} > 0. \quad (18)$$

Combining (9), (10), and (13) yields the no-arbitrage condition (see Appendix B)

$$r = \Gamma_Y(s)\dot{s} + \frac{\alpha\delta uL}{\Phi(s; \alpha)} - \mu. \quad (19)$$

The representative household satisfies the Euler equation $\frac{\dot{C}}{C} = r - \rho$. Using the resource constraint $Y = C + X$, no-arbitrage condition (19), and the Euler equation, we obtain the dynamics of u :

$$\frac{\dot{u}}{u} = A_1(s)\delta uL - A_2(s), \quad (20)$$

where

$$A_1(s) = \frac{\alpha}{\Phi(s; \alpha)} + 1 + s\Gamma_c(s) > 0, \quad A_2(s) = \Gamma_c(s)\{s\delta L - (1-s)\mu\} + \mu + \rho + \delta L.$$

Since $A_1(s) > 0$ for all $s \in [0, 1]$, the sign of \dot{u}/u is entirely governed by $A_2(s)$. If $A_2(s) < 0$, then $\dot{u}/u > 0$ for all $u \in (0, 1)$, so that the economy converges to the corner solution $u = 1$. In this case, $g_A = \delta(1-u)L = 0$, and the economy exhibits no sustained growth. Since our focus is on sustained positive-growth equilibria, we concentrate on the region of the state space where $A_2(s) > 0$. Technical derivations underlying equation (20) are reported in Appendix A and B.

3.2 Equilibrium and Steady States

For each given level of market competitiveness s , the stationary value of the jump variable u is uniquely determined by the condition $\dot{u} = 0$ in (20). This defines the static equilibrium allocation conditional on s :

$$\bar{u}(s) = \frac{\rho + \mu + \delta L + \Gamma_c(s)\{s\delta L - (1-s)\mu\}}{\left[\frac{\alpha}{\Phi(s; \alpha)} + 1 + s\Gamma_c(s)\right] \delta L}. \quad (21)$$

Substituting $\bar{u}(s)$ into the law of motion (16) yields a one-dimensional dynamic equation in the state variable s . Long-run outcomes are therefore characterized by the fixed points of this reduced system.

Definition (Steady state). A steady state is a pair (u^*, s^*) such that $\dot{u} = 0$ and $\dot{s} = 0$, with feasibility $u^* \in (0, 1]$ and $s^* \in [0, 1]$. We call $s^* = 1$ the competitive steady state \mathcal{C} , and any feasible $s^* \in (0, 1)$ an interior steady state \mathcal{M} .

At a steady state,

$$g_A = \delta(1 - u^*)L, \quad r = \rho + g_A.$$

Since $\dot{s} = 0$ in steady state implies $s\delta L = (1 - s)\mu$, the expression simplifies to

$$\bar{u}^*(s) = \frac{\rho + \mu + \delta L}{\left[\frac{\alpha}{\Phi(s; \alpha)} + 1\right] \delta L}. \quad (22)$$

3.3 Steady States and Dynamics of s

We now characterize the steady states of the economy. Substituting the stationary equilibrium allocation $\bar{u}(s)$ obtained in (21) into (16) yields a closed-form law of motion for market competitiveness:

$$\dot{s} = \frac{(1 - s)\mu\{\alpha + \Phi(s; \alpha)\} - s\{\alpha\delta L - \Phi(s; \alpha)(\mu + \rho)\}}{\alpha + \Phi(s; \alpha)\{1 + s\Gamma_c(s)\}}. \quad (23)$$

Since $\alpha > 0$, $\Phi(s; \alpha) > 0$, and $\Gamma_c(s) > 0$ for all $s \in [0, 1]$, the denominator of (23) is strictly positive. Hence, both the sign of \dot{s} and the local stability of any steady state are entirely determined by the numerator, or equivalently by the sign of $\Sigma'(s)$ at the root.

The reduced one-dimensional dynamics of s therefore reflects the opposing forces of innovation and imitation summarized by δ and μ .

Interior steady states satisfy $\dot{s} = 0$. Setting the numerator of (23) equal to zero yields the quadratic equation

$$\Sigma(s) = \Lambda\rho s^2 + s\{\Omega_1 - \alpha\delta L\} + (1 + \alpha)\mu, \quad (24)$$

where

$$\Lambda = \alpha^{-\frac{\alpha}{1-\alpha}} - 1 > 0, \quad \Omega_1 = \mu(\Lambda - \alpha) + \rho.$$

Importantly, $\Sigma(s)$ is a quadratic polynomial in s . The algebraic steps leading to the quadratic form are reported in Appendix C.

We restrict attention to trajectories in the admissible state space $[0, 1]$, which is invariant under the dynamics.

An interior steady state is a value $s^* \in (0, 1)$ such that $\dot{s} = 0$. Because the denominator of \dot{s} is strictly positive, this is equivalent to $\Sigma(s^*) = 0$.

At the boundary, stability is evaluated one-sidedly within the admissible set $[0, 1]$. In particular, the upper boundary $s = 1$ is locally attracting if $\dot{s} > 0$ for s sufficiently close to 1 from below. Since the denominator of (23) is strictly positive, the sign of \dot{s} is governed by the numerator, equivalently by $\Sigma(s)$.

Proposition 1 (Local stability at the competitive corner). *The competitive corner $s = 1$ is locally attracting if and only if $\Sigma(1) > 0$. Moreover,*

$$\Sigma(1) = (\Lambda + 1)(\rho + \mu) - \alpha\delta L, \quad \Lambda + 1 = \alpha^{-\frac{\alpha}{1-\alpha}},$$

so that

$$\Sigma(1) > 0 \iff \alpha^{\frac{1}{1-\alpha}}\delta L < \rho + \mu.$$

Remark (Boundary knife-edge). If $\Sigma(1) = 0$, then $\dot{s}(1) = 0$ and the boundary point is nonhyperbolic. In this knife-edge case, $s = 1$ coincides with the larger root of $\Sigma(s) = 0$ and can be interpreted as a boundary threshold rather than a locally attracting corner.

We now characterize the existence and uniqueness of interior steady states.

Lemma 1 (Uniqueness of the interior steady state when $\Sigma(1) < 0$). *Suppose $\Sigma(1) < 0$. Since $\Sigma(0) = (1 + \alpha)\mu > 0$ and $\Sigma(s)$ is continuous, there exists at least one root in $(0, 1)$.*

Moreover, because $\Sigma(s)$ is a convex quadratic polynomial (with $\Lambda\rho > 0$), there cannot be two distinct roots in $(0, 1)$ if $\Sigma(1) < 0$. Hence, there exists a unique interior root $s_1 \in (0, 1)$, and the second root (if real) must satisfy $s_2 > 1$.

Consequently, the competitive corner is unstable, and the economy converges to the unique interior steady state for any initial condition $s_0 \in (0, 1)$.

We next characterize the conditions under which multiple interior roots arise.

Lemma 2 (Real roots of Σ). *Let*

$$\Delta = (\Omega_1 - \alpha\delta L)^2 - 4\Lambda\rho(1 + \alpha)\mu.$$

If $\Delta > 0$, $\Sigma(s) = 0$ admits two distinct real roots $s_1 < s_2$.

Lemma 3 (Interior feasibility). *Let*

$$s^\dagger = \frac{\alpha\delta L - \Omega_1}{2\Lambda\rho}$$

denote the vertex of $\Sigma(s)$. If $\Delta > 0$ and $s^\dagger \in (0, 1)$, then $\Sigma(s^\dagger) < 0$. Since $\Sigma(0) > 0$ and $\Sigma(s)$ is convex, $\Sigma(s) = 0$ admits two real roots.

Moreover, if in addition $\Sigma(1) > 0$, both roots lie in $(0, 1)$.

Combining the previous lemmas yields the main result on equilibrium multiplicity.

Proposition 2 (Multiple steady states). *The economy admits multiple steady states if and only if*

$$\Delta > 0, \quad s^\dagger \in (0, 1), \quad \text{and} \quad \Sigma(1) > 0.$$

Since $\Sigma(s)$ is convex and the denominator of \dot{s} is positive, the smaller interior root is locally stable, whereas the larger root is unstable.

Accordingly, the smaller interior root corresponds to the steady state M , the larger root acts as a structural threshold, and the competitive corner $s = 1$ constitutes the steady state C .

Figure 1 illustrates the reduced one-dimensional dynamics of s in the coexistence region. The unstable root partitions the state space into two basins of attraction.

This threshold structure clarifies how regime selection emerges from the underlying state dynamics. The classification highlights that regime selection is governed by structural state dynamics rather than by expectation-based coordination. Multiplicity arises from the endogenous interaction between innovation and imitation through the evolution of market competitiveness. When the coexistence region obtains, small differences in initial conditions generate persistent divergence in long-run outcomes. The mechanism is entirely deterministic and driven by the nonlinear structure of the general equilibrium system.

The framework also carries clear policy implications. Because regime boundaries are characterized by threshold conditions, policy-induced changes in parameters such as imitation intensity or innovation incentives can shift the economy across regimes. When the economy operates near a threshold, even marginal parameter changes may lead to discontinuous shifts in long-run outcomes.

An example of an appearance region is depicted in Figure 2.

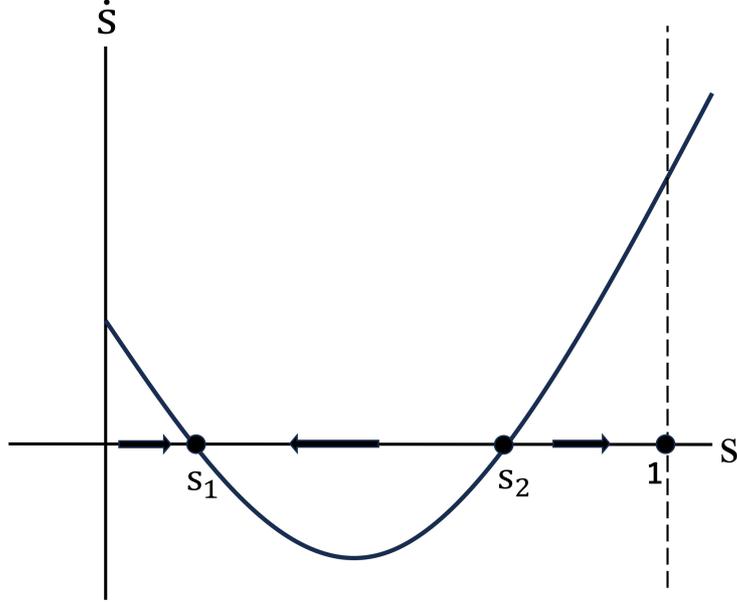


Figure 1: One-dimensional dynamics of market competitiveness. When two interior steady states $s_1 < s_2$ exist, s_1 is locally stable and s_2 is an unstable structural threshold. Arrows indicate the direction of motion.

Lemma 4. (Triple intersection of regime boundaries). *In the economically relevant region $\mu > 0$, the boundary $\Sigma(1) = 0$ intersects the boundary $s^\dagger = 1$ at a unique point*

$$\mu^* = \frac{\Lambda\rho}{1 + \alpha}.$$

Moreover, at this point the discriminant satisfies $\Delta = 0$. Hence the three regime boundaries $\Sigma(1) = 0$, $s^\dagger = 1$, and $\Delta = 0$ intersect at a unique point in parameter space.

Proof. See Appendix F.

As illustrated in Figure 2, the coexistence region is bounded by a linear stability condition, a quadratic discriminant condition, and the interiority condition $s^\dagger \in (0, 1)$, which intersect at a unique point μ^* in parameter space.

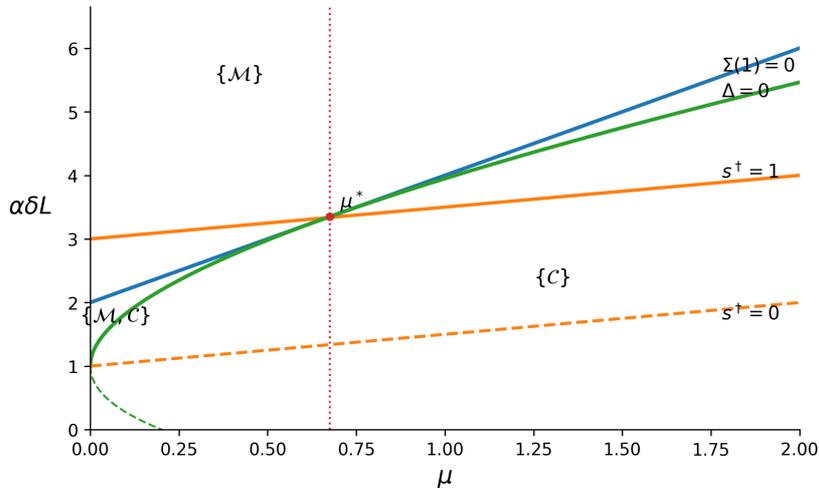


Figure 2: Regime boundaries in the $(\mu, \alpha\delta L)$ plane. The solid lines plot the binding boundaries $\Sigma(1) = 0$, $s^\dagger = 1$, and the upper branch of $\Delta = 0$. The thin dashed lines show non-binding extensions of $s^\dagger = 0$ and the lower branch of $\Delta = 0$. Region $\{\mathcal{M}\}$ (coexistence) is characterized by $\Sigma(1) > 0$, $\Delta > 0$, and $s^\dagger \in (0, 1)$, region $\{\mathcal{C}\}$ (competitive) by $\Sigma(1) < 0$, and region $\{\mathcal{M}, \mathcal{C}\}$ by parameter values for which both regimes are attainable depending on the initial competitiveness. The point μ^* denotes the unique triple intersection where $\Sigma(1) = 0$, $s^\dagger = 1$, and $\Delta = 0$ coincide (Lemma 4).

4 Conclusion

This paper has examined the dynamic interaction between innovation and imitation in a Romer-type growth model and its implications for the endogenous evolution of market structure. In the model, innovation creates monopolized sectors, while imitation gradually erodes monopoly power, generating endogenous changes in aggregate market competitiveness.

The analysis shows that this interaction generates nonlinear state dynamics in market competitiveness. Although the underlying general-equilibrium feedback operates through R&D investment decisions and household optimization, the steady-state condition reduces to a quadratic equation in the competitive share. This structure implies that, for certain parameter configurations, two interior roots may arise: a lower stable steady state and a higher unstable threshold. The unstable root separates the basins of attraction of distinct long-run regimes. In parameter space, the regime boundaries intersect at a unique point, organizing the geometry of coexistence and anchoring the threshold structure. When this configuration occurs, the economy may

converge either to a competitive steady state with zero entry or to an interior steady state with sustained growth, depending solely on its initial degree of competition.

Equilibrium selection therefore reflects the position of the economy relative to a structural threshold rather than expectation-based coordination. History dependence emerges because small differences in initial market structure can place the economy in different basins of attraction, leading to persistent divergence in growth regimes even under perfect foresight.

More broadly, the results highlight the importance of treating market competitiveness as an endogenous state variable in growth theory. The interaction between innovation and imitation does not merely determine growth rates; it shapes the evolution of market structure itself, which in turn governs regime selection.

Policy implications follow naturally from this threshold structure. Structural parameters such as imitation intensity or entry productivity determine the location of the unstable threshold. Policies that shift these parameters sufficiently can move the economy from a low-growth regime to a sustained-growth regime. By contrast, marginal interventions that alter current competitiveness without shifting the threshold may leave the economy within the same basin of attraction and therefore have limited long-run effects. The impact of competition or intellectual property policies is thus inherently non-linear and state-dependent.

Several extensions remain for future research. Endogenizing the imitation process or allowing for quality upgrading within previously competitive sectors would enrich the analysis and further clarify the structural forces governing market structure dynamics.

A Derivation of the Consumption Dynamics

From the resource constraint $Y = C + X$ and equations (7) and (8), we obtain

$$C = Y - X = \alpha^{\frac{2\alpha}{1-\alpha}} A\Phi(s; \alpha)L_Y - \alpha^{\frac{2}{1-\alpha}} A\Phi(s; 1)L_Y.$$

Dividing by Y yields the intermediate-input share

$$\chi(s) \equiv \frac{X}{Y} = \alpha^2 \frac{\Phi(s; 1)}{\Phi(s; \alpha)}. \quad (25)$$

Direct differentiation implies

$$\chi'(s) > 0 \quad \text{for } s \in (0, 1),$$

and

$$\chi(0) = \alpha^2, \quad \chi(1) = \alpha.$$

Since $\dot{\chi}(s) = \chi'(s)\dot{s}$, it is convenient to define

$$\Gamma_c(s) \equiv \frac{\chi'(s)}{1 - \chi(s)}.$$

Differentiating $C = (1 - \chi(s))Y$ with respect to time yields

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} - \Gamma_c(s)\dot{s}. \quad (26)$$

Straightforward differentiation implies

$$\Gamma_c(s) > 0, \quad \Gamma'_c(s) < 0.$$

From the Euler equation $\frac{\dot{C}}{C} = r - \rho$, the dynamics C given in (26) and the dynamics Y , given in (17), we obtain

$$r - \Gamma_Y(s)\dot{s} = \rho + \delta(1 - u)L + \frac{\dot{u}}{u} - \Gamma_c(s)\dot{s}. \quad (27)$$

B Derivation of the Labor Dynamics

Combining (8), (9), (10), and (13) yields

$$r = \frac{\dot{\tilde{V}}}{\tilde{V}} + \frac{\alpha\delta uL}{\Phi(s; \alpha)} - \mu$$

Using (13) together with the definition of $\Gamma_Y(s)$ in (18), which is derived from $\dot{\Phi}(s)/\Phi(s) = \Gamma_Y(s)\dot{s}$, the above equation is made into

$$r - \Gamma_Y(s)\dot{s} = \frac{\alpha\delta uL}{\Phi(s; \alpha)} - \mu.$$

Eliminating $r - \Gamma_Y(s)\dot{s}$ from this equation by using Eq. (27), we have

$$\frac{\alpha\delta uL}{\Phi(s; \alpha)} - \mu = \rho + \delta(1 - u)L + \frac{\dot{u}}{u} - \Gamma_c(s)\dot{s}.$$

Substituting the dynamics of s obtained as (16) into the above and rearranging terms yields

$$\frac{\dot{u}}{u} = \left[\frac{\alpha}{\Phi(s; \alpha)} + 1 + s\Gamma_c(s) \right] \delta uL - [\Gamma_c(s)\{s\delta L - (1 - s)\mu\} + \mu + \rho + \delta L].$$

Defining $A_1(s)$ and $A_2(s)$ gives equation (20).

C Reduction to a Quadratic Polynomial

Substituting the equilibrium allocation $u(s)$ obtained in (21) into (16) yields

$$\dot{s} = \frac{(1-s)\mu\{\alpha + \Phi(s; \alpha)\} - s\{\alpha\delta L - \Phi(s; \alpha)(\mu + \rho)\}}{\alpha + \Phi(s; \alpha)\{1 + s\Gamma_c(s)\}} = \frac{N(s)}{D(s)}.$$

Since $D(s) > 0$ for all $s \in [0, 1]$, steady states satisfy $N(s) = 0$. Expanding the numerator $N(s)$ and substituting the expression for $\Phi(s; \alpha)$ yields the quadratic polynomial

$$\Sigma(s) = \Lambda\rho s^2 + s\{\Omega_1 - \alpha\delta L\} + (1 + \alpha)\mu,$$

where

$$\Lambda = \alpha^{-\frac{\alpha}{1-\alpha}} - 1, \quad \Omega_1 = \mu(\Lambda - \alpha) + \rho.$$

Since $\alpha \in (0, 1)$, we have $\Lambda > 0$. Hence, the existence and multiplicity of interior steady states are fully characterized by the discriminant of $\Sigma(s)$.

D Proofs

Proof of Proposition 1. Since $\dot{s} = \Sigma(s)/D(s)$ with $D(s) > 0$, we have

$$\frac{d}{ds}\dot{s} = \frac{\Sigma'(s)D(s) - \Sigma(s)D'(s)}{D(s)^2}.$$

At a steady state where $\Sigma(s) = 0$, this reduces to

$$\frac{d}{ds}\dot{s} = \frac{\Sigma'(s)}{D(s)}.$$

Hence stability depends on the sign of $\Sigma'(1)$.

Proof of Proposition 2. The quadratic polynomial $\Sigma(s)$ admits two distinct real roots if and only if the discriminant

$$\Delta = (\Omega_1 - \alpha\delta L)^2 - 4\Lambda\rho(1 + \alpha)\mu$$

is strictly positive.

Moreover, since $\Lambda\rho > 0$, the vertex of $\Sigma(s)$ is given by

$$s^\dagger = \frac{\alpha\delta L - \Omega_1}{2\Lambda\rho},$$

and

$$\Sigma(s^\dagger) = -\frac{\Delta}{4\Lambda\rho}.$$

Thus $\Delta > 0$ is equivalent to $\Sigma(s^\dagger) < 0$.

Since $\Sigma(0) > 0$ and $\Sigma(s)$ is convex, $\Sigma(s^\dagger) < 0$ implies the existence of two real roots. The interior feasibility condition follows from $s^\dagger \in (0, 1)$ and $\Sigma(1) > 0$.

Proof of Lemma 4 (Threshold dynamics). Since $\Sigma(s)$ is convex and $D(s) > 0$, the sign of \dot{s} is determined by $\Sigma(s)$.

When two interior roots exist, $\Sigma'(s_1) < 0$ and $\Sigma'(s_2) > 0$. Hence s_1 is locally stable and s_2 unstable. The global dynamics follow from the one-dimensional phase diagram.

E Relative Positions of $\bar{u}(s)$ and $\bar{u}^*(s)$

To facilitate the analysis of the dynamics of $\bar{u}(s)$ along changes in s , we introduce an auxiliary function $\bar{v}(s)$.

$$\bar{v}(s) = \frac{\rho + \mu + \delta L + s\Gamma_c(s)\delta L}{\left[\frac{\alpha}{\Phi(s;\alpha)} + 1 + s\Gamma_c(s)\right] \delta L}. \quad (28)$$

Since $\bar{v}(s)$ is obtained by adding the same positive term ($s\Gamma_c(s)\delta L$) to both the numerator and denominator of $\bar{u}^*(s)$, the relative magnitude of $\bar{v}(s)$ and $\bar{u}^*(s)$ is entirely determined by whether $\bar{u}^*(s)$ is above or below unity.

$$\bar{u}^*(s) > 1 \iff \bar{u}^*(s) > \bar{v}(s), \quad \bar{u}^*(s) < 1 \iff \bar{u}^*(s) < \bar{v}(s).$$

Moreover, $\bar{v}(s) \geq \bar{u}(s)$ holds for all $s \in [0, 1]$, with equality only at $s = 1$. At the boundaries, we have $\bar{u}^*(0) = \bar{v}(0) > \bar{u}(0)$ and $\bar{u}(1) = \bar{v}(1) > \bar{u}^*(1)$. It should be emphasized that $u = \bar{u}(s)$ does not describe the transition dynamics of u or s ; it is merely the stationary allocation conditional on s .

Fig.fig:3) depicts these $\bar{u}^*(s)$, \bar{v} , and $\bar{u}(s)$.

F Proof of Lemma 4.(Triple Intersection)

We show that the three regime boundaries $\Sigma(1) = 0$, $s^\dagger = 1$, and $\Delta = 0$ intersect at a unique point in the economically relevant region $\mu > 0$.

Step 1. Intersection of $\Sigma(1) = 0$ and $s^\dagger = 1$

The boundary condition

$$\Sigma(1) = 0$$

is equivalent to

$$\alpha\delta L = (\Lambda + 1)(\rho + \mu). \quad (\text{E.1})$$

The condition $s^\dagger = 1$ implies

$$\alpha\delta L = \Omega_1 + 2\Lambda\rho,$$

where

$$\Omega_1 = \rho + \mu(\Lambda - \alpha).$$

Hence

$$\alpha\delta L = (1 + 2\Lambda)\rho + (\Lambda - \alpha)\mu. \quad (\text{E.2})$$

Equating (E.1) and (E.2) yields

$$(\Lambda + 1)(\rho + \mu) = (1 + 2\Lambda)\rho + (\Lambda - \alpha)\mu.$$

Rearranging,

$$-\Lambda\rho + (1 + \alpha)\mu = 0,$$

so that

$$\mu^* = \frac{\Lambda\rho}{1 + \alpha}. \quad (\text{E.3})$$

Since $\Lambda > 0$ and $\rho > 0$, equation (E.3) determines a unique positive value $\mu^* > 0$.

Thus the two boundaries intersect at a unique positive value $\mu^* > 0$.

Step 2. Verification that $\Delta = 0$ at μ^*

The discriminant is given by

$$\Delta = (\Omega_1 - \alpha\delta L)^2 - 4\Lambda\rho(1 + \alpha)\mu.$$

Under the condition $s^\dagger = 1$, we have

$$\alpha\delta L = \Omega_1 + 2\Lambda\rho,$$

so that

$$\Omega_1 - \alpha\delta L = -2\Lambda\rho.$$

Hence

$$\Delta = (2\Lambda\rho)^2 - 4\Lambda\rho(1 + \alpha)\mu = 4\Lambda\rho(\Lambda\rho - (1 + \alpha)\mu).$$

Substituting $\mu = \mu^*$ from (E.3),

$$\Delta = 4\Lambda\rho(\Lambda\rho - \Lambda\rho) = 0.$$

Therefore, the three regime boundaries $\Sigma(1) = 0$, $s^\dagger = 1$, and $\Delta = 0$ intersect at a unique point in parameter space. This completes the proof of the triple intersection property.

Acknowledgement I am especially grateful to Yasunobu Tomoda and Tatsuro Isaisako for their beneficial comments. This work was conducted with financial support from JSPS KAKENHI Grant Number 15K03360, JSPS KAKENHI Grant Number 26380348, and the grant from Kobe Academic Park Association for the Promotion of Inter-University Research and Exchange.

References

- [1] Aghion, P., Cherif, R., and Hasanov, F. (2021) Competition, Innovation, and Inclusive Growth (IMF Working Paper WP/21/80; 2021).
- [2] Iwaisako, T, Futagami, K, (2003) Patent policy in an endogenous growth model. *Journal of Economics* 78:239–258. <https://doi.org/10.1007/s00712-002-0541-1>
- [3] Kuwahara, S (2007) The mechanics of economic growth through capital accumulation and technological progress. *Japan Economic Review* 58:504–523.
- [4] Matsuyama, K, (1999) Growing through cycles. *Econometrica* 67:335–347. <https://doi.org/10.1111/1468-0262.00021>
- [5] Peters, Michael. (2020). Heterogeneous Markups, Growth, and Efficiency. *Econometrica*, 88 (5): 2037-2081.
- [6] Romer, P. M. (1990) Endogenous technological change. *Journal of Political Economy* 98:S71–S102.

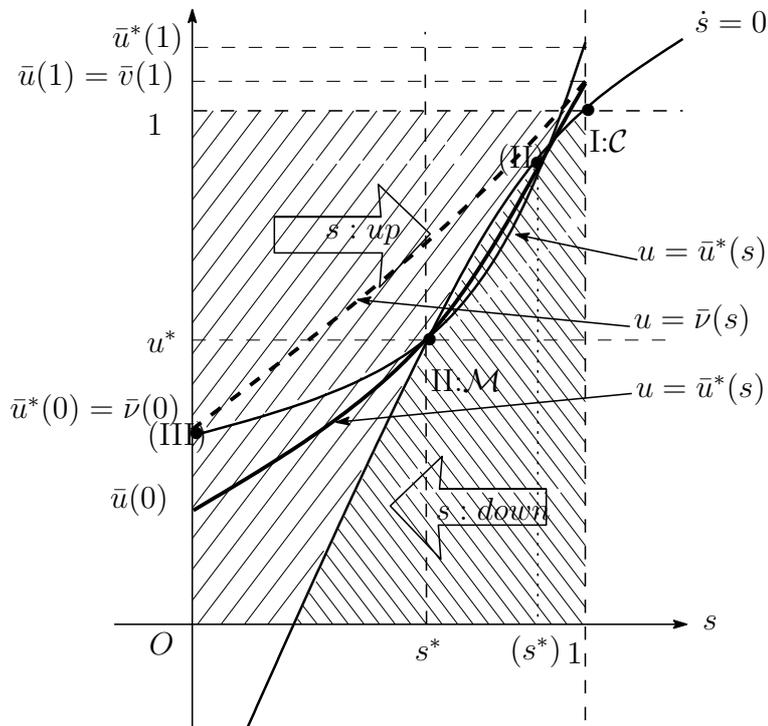
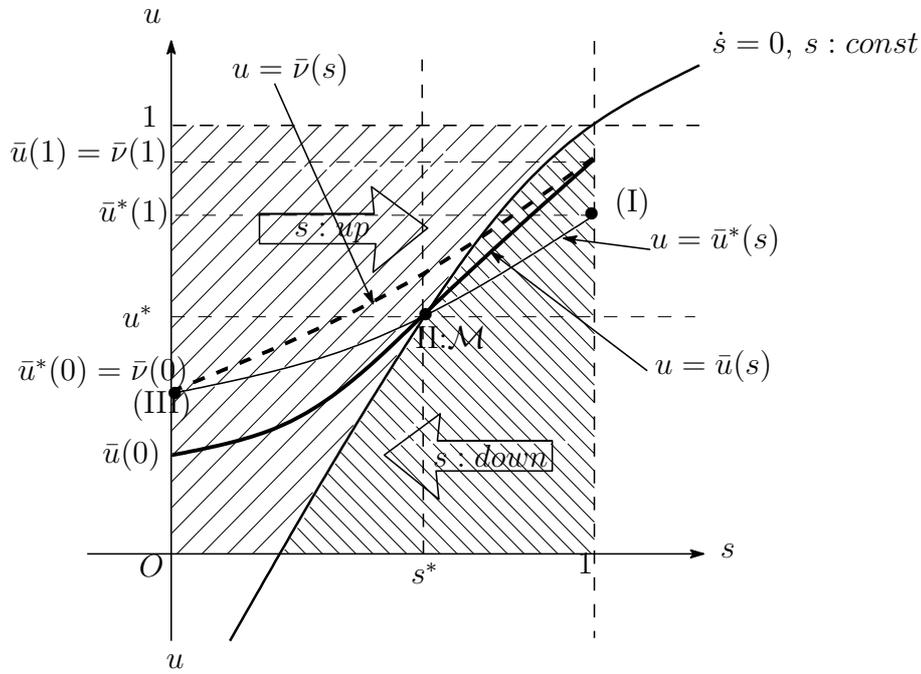


Figure 3: Steady state on the $(s - u)$ plain