Growth with Perishable Knowledge: Endogenous Supply of Innovative Capacity*

Shiro Kuwahara†

February, 16, 2026

Abstract

This paper develops an endogenous growth model with perishable knowledge and an endogenous supply of innovative capacity. Departing from conventional formulations in which innovative inputs accumulate as durable stocks, knowledge-intensive labor is modeled as a per-period flow whose productive relevance must be continuously regenerated. Within an R&D-based framework, the supply of effective innovative input is determined jointly by education effort and the allocation of regenerated knowledge to research.

In this environment, long-run growth is not guaranteed by accumulation, but depends on the feasibility of sustaining regeneration and innovation. We show that an interior balanced growth path with positive innovation exists if and only if a threshold condition is satisfied: the returns to knowledge regeneration must exceed the rate of discounting. If this condition is not satisfied, the interior innovation regime ceases to exist and the unique equilibrium converges to a boundary allocation with zero R&D activity and zero per-capita growth. Long-run growth is therefore determined by equilibrium feasibility rather than by marginal variations of a pre-existing balanced growth path.

Keywords: endogenous growth; R&D; human capital; knowledge depreciation; poverty traps

JEL Classification: O41, O31, J24

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*I would like to thank Katsuhiko Hori and Yuho Yamashita for their helpful comments. All remaining errors are the sole responsibility of the author.

†*Affiliation:* University of Hyogo, *E-mail:* kuwahara@em.u-hyogo.ac.jp

1 Introduction

Recent decades have witnessed rapid technological progress and an expanding role of knowledge-intensive industries. In many advanced sectors, maintaining productivity increasingly requires continuous updating of skills. A growing empirical literature documents the limited durability of skills in technologically dynamic environments. Weinberg (2002) shows that the introduction of new computing technologies erodes the returns to prior experience. Kambourov and Manovskii (2009) document the occupational specificity of human capital and substantial wage losses upon occupational switching. More recently, Deming and Noray (2020) show that earnings premia in STEM careers decline over the life cycle, consistent with rapid skill depreciation. Theoretically, the idea that human capital facilitates technological adoption dates back to Nelson and Phelps (1966).

Motivated by these observations, this paper studies an economy in which knowledge becomes obsolete unless continuously regenerated. Educated labor is produced through education and learning, but its productive relevance does not accumulate automatically over time. Instead, knowledge-intensive labor must be regenerated each period. While extreme, this assumption isolates a mechanism implicit in rapidly innovating sectors: growth depends not only on how much knowledge is accumulated, but on whether technologically relevant knowledge can be continuously reproduced.

To formalize this idea, the model builds on the R&D-based growth framework of Romer (1990), but departs from the conventional treatment of innovative inputs as durable. Knowledge-intensive labor is modeled as a perishable flow rather than a stock, so that innovation depends on per-period regeneration rather than past accumulation. In this framework, the supply of innovative capacity is determined endogenously through the joint interaction between education effort and the allocation of regenerated knowledge to R&D. Unlike standard R&D-based growth models, in which innovative inputs are either taken as exogenously given or accumulate mechanically as durable stocks, the present model derives the effective supply of R&D input from equilibrium behavior. As a result, sustained growth is not automatic: it emerges only when a feasibility condition is satisfied. This implies that long-run growth is governed by a parameter-dependent existence condition rather than by smooth comparative-static adjustments within a given regime.

The analysis delivers three main results.

First, long-run growth depends on the equilibrium feasibility of regeneration and innovation rather than on the accumulation of a durable human-capital stock.

Second, sustained growth exists if and only if a feasibility condition is satisfied. An interior innovation regime arises precisely when the equilibrium supply of effective innovative capacity is sufficiently high relative to discounting, characterized by the condition $\Gamma(1) > \rho$. Within the interior region, growth varies continuously with fundamentals. However, when this condition fails, the interior regime ceases to exist and the economy converges to a boundary allocation with zero R&D activity. Long-run growth is therefore governed by equilibrium feasibility rather than by smooth marginal adjustments within a given regime.

Third, the model highlights two distinct welfare margins: a static distortion arising from monopoly pricing, and a dynamic margin associated with the feasibility of sustained innovation. While the former is standard in R&D-based growth models, the latter emerges from the endogenous supply of innovative capacity.

The present approach relates to three strands of the growth literature. The first is the R&D-based framework initiated by Romer (1990) and extended by Aghion and Howitt (1992), where long-run growth depends on innovative effort. In these models, however, the supply of innovative input is typically treated as either exogenously given or as accumulating as a durable stock. The second strand is the human-capital accumulation literature, including Lucas (1988) , which models skill formation as the intertemporal buildup of human capital.[1]

While these models endogenize education effort, they maintain the durability of accumulated knowledge. The third concerns knowledge obsolescence and technological adaptation, as in Nelson and Phelps (1966) and subsequent work on skill depreciation. In contrast, this paper integrates endogenous education and R&D allocation within a framework in which effective innovative capacity is supplied endogenously and its marginal productivity remains bounded even when R&D effort approaches zero. As a result, sustained innovation becomes a feasibility condition rather than an automatic outcome, and long-run growth is governed by the existence of an interior innovation regime.

Section 2 presents the model. Section 3 characterizes equilibrium dynamics and establishes the threshold condition for sustained growth. Section 4 discusses welfare implications and the role of endogenous innovative capacity. Section 5 concludes.

2 The Model

We extend an R&D-based growth framework in the spirit of Romer (1990) by modeling knowledge-intensive labor as perishable. Each period, households allocate time between raw labor supply and education/learning, which generates a flow of perishable knowledge labor. This flow is allocated between final-goods production and R&D. Previously acquired skills do not carry over unless regenerated.

[1] See Ben-Porath (1967) for an early formal model of human capital accumulation.

The economy consists of three sectors: a final-goods sector (Y), a continuum of monopolistically competitive intermediate-goods firms indexed by $i \in [0, A]$, and an R&D sector that expands the variety frontier $\dot{A}$. There are three productive inputs: raw labor L, technological knowledge A, and perishable knowledge labor H. The key departure from standard formulations is that H is a per-period flow generated through education, rather than a durable stock that accumulates over time.

2.1 Goods Production

Final output is produced using raw labor, perishable knowledge labor allocated to final production (H_Y), and a continuum of intermediate inputs:

$$Y = \left(L^{\beta} H_Y^{(1-\beta)}\right)^{1-\alpha} \int_0^A \tilde{X}(i)^{\alpha} di, \quad 0 < \alpha, \beta < 1. \tag{1}$$

where $\tilde{X}(i)$ denotes the input of intermediate good i.

The first-order conditions imply

$$w_L = \frac{\beta(1-\alpha)Y}{L}, \quad w_Y = \frac{(1-\beta)(1-\alpha)Y}{H_Y}, \quad p(i) = \alpha(L^{\beta} H_Y^{1-\beta})^{1-\alpha} \tilde{X}(i)^{\alpha-1}. \tag{2}$$

Following Romer (1990), one unit of intermediate goods requires η units of final goods. In symmetric equilibrium,

$$\tilde{X} = \left(\frac{\alpha^2}{\eta}\right)^{\frac{1}{1-\alpha}} L^{\beta} H_Y^{1-\beta}, \quad p = \frac{\eta}{\alpha}. \tag{3}$$

Aggregate output becomes

$$Y = \left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} A L^{\beta} H_Y^{1-\beta}. \tag{4}$$

Resource constraints imply

$$X = \alpha^2 Y, \quad C = (1-\alpha^2)Y. \tag{5}$$

where $X \equiv \int_0^A \tilde{X}(i)\, di$ denotes aggregate intermediate input and C denotes aggregate consumption.

2.2 The R&D Sector

The R&D sector expands the technology frontier according to

$$\dot{A} = \delta \frac{A H_A}{\Omega}, \tag{6}$$

where H_A denotes perishable knowledge labor allocated to R&D, and Ω captures research difficulty, following the formulation in Segerstrom (1998).[2]

Free entry implies that the value of a patent $\tilde{V}$ satisfies

$$r\tilde{V} = \dot{\tilde{V}} + \tilde{\Pi}^M. \tag{7}$$

Positive R&D activity requires

$$\delta \frac{\tilde{V} A}{\Omega} = w_A, \tag{8}$$

which equates the marginal return to innovation with the wage paid to perishable knowledge labor in R&D.

2.3 Households

The representative household supplies one unit of time. A fraction $l \in [0, 1]$ is supplied as raw labor, and the remaining time is devoted to education. Education generates per-period perishable knowledge labor:

$$h = \Psi(l), \quad \Psi'(\cdot) < 0. \tag{9}$$

Preferences are given by

$$U = \int_0^\infty \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt, \quad \rho > 0, \ \sigma > 0, \tag{10}$$

where c denotes per-capita consumption.

The Euler equation is

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r - n - \rho). \tag{11}$$

[2] Alternative specifications, including the semi-endogenous growth framework of Jones (1995), are fully consistent with long-run growth determination. However, many formulations impose an Inada-type condition under which the marginal productivity of R&D input diverges as $H_A \to 0$, ensuring positive R&D activity for arbitrarily small input levels. Such a specification would preclude the possibility of a no-R&D equilibrium. To allow for regime non-existence—which is central to the analysis of regeneration feasibility—the present paper adopts a Romer-Segerstrom type formulation in which the marginal productivity of R&D remains bounded. This boundedness property becomes crucial in deriving the equilibrium condition that governs whether sustained innovation is feasible.

We specify the education technology as

$$h = b(1 - l)^{\phi}, \quad b > 0, \ \phi \in (0, 1). \tag{12}$$

This specification implies that knowledge-intensive labor is generated anew each period and does not accumulate across periods.

Because perishable knowledge labor can be allocated either to final-goods production or to R&D, define

$$u := \frac{H_Y}{H},$$

so that $1 - u = \frac{H_A}{H}$. Let population size be denoted by N, growing at the exogenous rate

$$\dot{N} = nN.$$

Aggregate variables are related to per-capita variables by

$$L = Nl, \quad H = Nh, \quad C = Nc.$$

Thus, individual allocations (l, h, c) determine aggregate inputs used in production. Since $H = Nh$, the effective supply of knowledge labor is endogenous and depends on individual education choices. Solving the household's optimality conditions yields

$$l(u) = \frac{\beta u}{\beta u + \phi(1 - \beta)}, \quad h(u) = b \left(\frac{\phi(1 - \beta)}{\beta u + \phi(1 - \beta)} \right)^{\phi}. \tag{13}$$

This allocation variable will play a central role in the analysis below.

2.4 Growth and Allocation

Using $H_A = (1 - u)H$, the growth rate of technology is

$$g_A(u) = \delta(1 - u)h(u). \tag{14}$$

An increase in R&D allocation (a decrease in u) raises g_A.

3 Dynamics and Steady States

This section characterizes equilibrium dynamics and the conditions under which sustained growth is feasible. Because perishable knowledge labor is allocated between final production and R&D, the evolution of the economy can be summarized by the dynamics of the allocation variable u.

3.1 Dynamic Equation

Using the free-entry condition (8) together with the expressions for wages and profits, the value of a patent can be written as

$$\tilde{V} = \frac{(1-\beta)(1-\alpha)}{\delta} \left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} N\, l(u)^{\beta} \big(uh(u)\big)^{-\beta}. \tag{15}$$

Differentiating (15) and using the arbitrage condition (7) yields

$$\frac{\dot{\tilde{V}}}{\tilde{V}} = n + \Phi_1 \frac{\dot{u}}{u} = r - \frac{\alpha\delta}{1-\beta} uh(u), \tag{16}$$

where Φ_1 collects the effect of changes in the allocation variable u on the value of innovation.

To streamline notation, define the (allocation) elasticities of labor supplies with respect to u as

$$\varepsilon_{ul} := \frac{l'(u)u}{l(u)}, \qquad \varepsilon_{uh} := -\frac{h'(u)u}{h(u)}.$$

Using the closed-form expressions in (13), these elasticities satisfy

$$\varepsilon_{ul} = \frac{\phi(1-\beta)}{\beta u + \phi(1-\beta)} \in (0,1), \qquad \varepsilon_{uh} = \frac{\beta\phi u}{\beta u + \phi(1-\beta)} \in (0,1), \quad \text{for all } u \in (0,1).$$

Therefore,

$$\Phi_1 = \beta\left[\varepsilon_{ul} - 1 + \varepsilon_{uh}\right] = -\frac{\beta^2 u(1-\phi)}{\beta u + \phi(1-\beta)} < 0. \tag{17}$$

Combining the Euler equation (11) with the balanced-growth implication $g_c = g_A = \delta(1-u)h(u)$ gives

$$\Phi_2 \frac{\dot{u}}{u} = r - n - \rho - \delta(1-u)h(u), \tag{18}$$

where

$$\Phi_2 = \sigma\left[\beta\varepsilon_{ul} + (1-\beta)\left(1 + \varepsilon_{uh}\right)\right] > 0. \tag{19}$$

Eliminating $r - n$ from (16) and (18) yields the law of motion for u:

$$(-\Phi_1 + \Phi_2)\frac{\dot{u}}{u} = \Gamma(u) - \rho, \tag{20}$$

where

$$\Gamma(u) = \left[\left(\frac{\alpha}{1-\beta} + \sigma \right) u - \sigma \right] \delta h(u). \tag{21}$$

The coefficient $-\Phi_1 + \Phi_2$ is strictly positive. Intuitively, $\Phi_1 < 0$ reflects how a shift in u affects the private return to innovation through wages and profits, whereas $\Phi_2 > 0$ captures the intertemporal substitution channel from household optimization. As a result, the direction of motion of u is governed solely by the sign of $\Gamma(u) - \rho$.

3.2 Steady States and Growth

A steady state requires $\dot{u} = 0$, which implies

$$\Gamma(u^*) = \rho.$$

Because $\Gamma'(u) > 0$ on $(0, 1)$ and $\Gamma(0) < 0$, and since the feasible set is $u \in [0, 1]$, an interior steady state exists if and only if

$$\Gamma(1) > \rho.$$

In addition, the transversality condition must be satisfied. Using the Euler equation and the balanced-growth relation $g_c = g_A = \delta(1-u)h(u)$, the transversality condition requires

$$\rho > (1-\sigma)\delta(1-u^*)h(u^*).$$

Following Hall (1988), we assume $\sigma > 1$. Under this assumption, the right-hand side is negative, so the transversality condition is automatically satisfied. Hence, the feasibility of sustained growth is fully characterized by the condition $\Gamma(1) > \rho$.

Given a steady-state allocation u, the technology growth rate is

$$g_A(u) = \delta(1-u)h(u).$$

Because output grows proportionally with A in this framework, the per-capita growth rate satisfies

$$g_y(u) = g_A(u).$$

Proposition 3.1 (Interior Innovation Regime) *If $\Gamma(1) > \rho$, there exists a unique interior steady state $u^* \in (0, 1)$ satisfying $\Gamma(u^*) = \rho$. Because u is forward-looking, rational expectations imply that the economy jumps immediately to u^*. In this equilibrium, per-capita* *growth is strictly positive and given by*

$$g_y^* = \delta(1 - u^*)h(u^*).$$

Proposition 3.2 (No-R&D Regime) *If $\Gamma(1) < \rho$, there exists no interior solution to $\Gamma(u) = \rho$. The unique steady state is the boundary allocation $u = 1$, so that $H_A = 0$ and*

$$g_y^* = 0.$$

4 Endogenous Innovative Capacity and Growth Regimes

The analysis highlights three structural implications of growth in an economy with perishable knowledge labor.

4.1 Growth as Regeneration Rather than Accumulation

In many R&D-based growth models, long-run growth is driven by the accumulation of a durable stock of human capital. In contrast, the present framework replaces accumulation with regeneration. Because previously acquired knowledge becomes fully obsolete unless updated, the effective input into innovation is not a stock variable but a per-period flow of regenerated knowledge labor.

Consequently, the steady-state growth rate

$$g_y^* = \delta(1 - u^*)h(u^*)$$

depends directly on the amount of knowledge regenerated and allocated to R&D in each period. Growth is therefore sustained only if the economy continuously reproduces technologically relevant knowledge. In this sense, growth is a contemporaneous outcome of regeneration effort rather than an intertemporal consequence of past accumulation.

4.2 Endogenous Innovative Capacity and Regime Feasibility

Although the equilibrium condition can be expressed in a reduced form resembling the standard asset-return versus discounting trade-off in Romer-type models, the underlying structure differs in an important way. In the Romer framework, the effective R&D input is typically tied to an exogenously given stock of human capital or labor endowment. In contrast, in the present model the effective innovative input $H_A = (1-u)H$ depends on the endogenous regeneration of perishable knowledge labor. The total supply of effective knowledge labor $H = Nh(u)$ is itself determined by education choices and interacts with the allocation variable u.

The model therefore endogenizes not only the allocation of innovative effort but also the effective supply of innovative capacity itself. As a result, the threshold condition $\Gamma(1) > \rho$ governs not merely the growth rate, but the feasibility of sustaining an interior innovation regime.

4.3 Threshold Structure and Regeneration-Based Poverty Traps

Propositions 3.1 and 3.2 establish a clear threshold condition for sustained growth: an interior balanced growth path exists if and only if

$$\Gamma(1) > \rho.$$

Unlike many endogenous growth models in which an interior balanced-growth allocation exists broadly and parameters primarily affect the growth rate within that regime, the present framework features a threshold in equilibrium feasibility. Within the interior region, both u^* and the implied growth rate vary continuously with fundamentals. However, when parameters cross the boundary defined by $\Gamma(1) = \rho$, the interior regime ceases to be feasible and the economy selects the boundary allocation with zero R&D.

The stagnation outcome differs from conventional poverty traps driven by insufficient physical or human capital stocks. Here, stagnation arises not because accumulated resources are low, but because the regeneration of knowledge-intensive labor is no longer privately sustainable. Even if the economy possesses a high level of technological knowledge at a given date, growth cannot continue unless sufficient updating effort is undertaken each period. The model thus identifies a regeneration-based poverty trap. More broadly, it highlights two distinct welfare margins: a static distortion arising from monopoly pricing, and a dynamic margin associated with the feasibility of sustained innovation. While the former is present in standard R&D-based growth models, the latter emerges from the interaction between knowledge regeneration and innovation incentives.

4.4 Welfare and Regeneration Feasibility

The decentralized equilibrium inherits the standard monopoly distortion of the R&D-based growth model. Intermediate-goods producers charge a markup, reducing contemporaneous output relative to the social optimum. In an interior innovation regime, this static inefficiency is accompanied by sustained long-run growth, generating the familiar trade-off between static losses and dynamic gains.

The present framework adds a second margin of welfare relevance. Sustained growth is feasible only when the joint equilibrium condition $\Gamma(1) > \rho$ holds. This condition reflects the interaction between the supply of effective knowledge through education and the incentives to allocate that knowledge to R&D.

When this threshold fails, the economy converges to a no-R&D equilibrium with zero per-capita growth. Welfare losses may then arise not only from the static monopoly distortion, but also from the disappearance of the interior innovation regime itself. Because regime feasibility depends on equilibrium interactions between education and R&D profitability, distortions affecting either margin can have amplified welfare consequences by ruling out the interior innovation regime.

5 Conclusion

This paper develops an endogenous growth model with perishable knowledge, adopting a regeneration-based approach to innovative activity. Departing from the conventional treatment of human capital as a durable stock, knowledge-intensive labor is modeled as a per-period flow whose productive relevance must be continuously renewed. Within an R&D-based framework, sustained growth depends on the economy 's ability to regenerate technologically relevant skills.

The central result is that an interior balanced growth path with positive innovation exists if and only if

$$\Gamma(1) > \rho,$$

that is, when the returns to knowledge regeneration exceed the rate of discounting. Long-run growth is therefore governed by regeneration effort rather than by stock accumulation. If this condition is not satisfied, the unique equilibrium is a boundary allocation with zero R&D activity and zero per-capita growth.

The framework is intentionally stylized. By modeling full obsolescence as a limiting case, the paper isolates the structural implications of regeneration-based growth in environments where knowledge depreciates rapidly. The analysis shows how the durability of knowledge shapes the feasibility of sustained innovation.

Several extensions remain for future research. Allowing for partial depreciation or combining accumulation and regeneration within a unified structure would generate richer dynamics while preserving the core mechanism identified here. Such extensions would also permit a fuller welfare analysis, distinguishing more precisely between static distortions associated with monopoly pricing and dynamic losses arising from the disappearance of the interior innovation regime. Embedding regeneration feasibility into a broader policy framework may further clarify how education and innovation incentives jointly shape long-run welfare outcomes.

Appendix A. Comparison with the Standard Romer Model

For comparison, we restate the steady-state growth rate in the original Romer (1990) framework using notation consistent with the present paper.

In the Romer model, the balanced-growth rate can be written as

$$g^* = \frac{\delta H - \frac{1-\beta}{\alpha}\rho}{\theta\frac{1-\beta}{\alpha} + 1}, \tag{22}$$

where H denotes the stock of human capital.

Positive growth requires

$$\delta H - \frac{1-\beta}{\alpha}\rho > 0.$$

Differentiating (22) yields the familiar comparative statics:

$$\frac{\partial g^*}{\partial \delta} > 0, \quad \frac{\partial g^*}{\partial H} > 0, \quad \frac{\partial g^*}{\partial \alpha} > 0, \quad \frac{\partial g^*}{\partial \beta} > 0, \quad \frac{\partial g^*}{\partial \rho} < 0.$$

In contrast, in the present model growth depends on the regeneration of perishable knowledge labor rather than on the level of a durable stock. Sustained growth requires the existence condition $\Gamma(1) > \rho$, which governs whether an interior innovation regime is feasible.

Appendix B. Threshold Condition and Comparative Statics

This appendix derives the parameter dependence of the threshold condition for sustained growth.

Recall that sustained growth requires

$$\Gamma(1) > \rho,$$

which is equivalent to

$$\delta h(1) > \frac{1-\beta}{\alpha}\rho. \tag{23}$$

Using

$$h(1) = b\left(\frac{\phi(1-\beta)}{\beta+\phi(1-\beta)}\right)^{\phi},$$

condition (23) can be written as

$$\delta b > \frac{1-\beta}{\alpha}\rho\left(\frac{\beta+\phi(1-\beta)}{\phi(1-\beta)}\right)^{\phi}. \tag{24}$$

Comparative Statics

From (24):

- Higher education efficiency b enlarges the interior region.
- Higher innovation efficiency δ enlarges the interior region.
- A higher discount rate ρ shrinks the interior region.
- A larger curvature parameter ϕ reduces $h(1)$ and thus makes sustained growth harder to sustain.
- A larger β increases R&D profitability and enlarges the interior region.

These results formalize the parameter effects summarized in the text.

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