Management Ability, Long-run Growth, and Poverty Traps

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Abstract

This study establishes an R&D-based growth model that includes the functional difference between labor and human capital in the production of goods. In our analysis, the human capital is used by the managers in the manufacturing process. Such an allocation of human capital yields three possible steady states: endogenous growth, poverty traps, and multiple equilibria. Economies are sorted into these steady states according to the endowments of labor, human capital, and knowledge. Thus, the obtained steady states explain some economic growth patterns, such as polarization and leapfrogging of economies.

Keywords: Management ability, R&D, multiple equilibria, poverty traps, long-run growth

JEL Classification: E00, O00, O41.

1 Introduction

Easterly (1994) and Quah (1996, 1997) provide evidence that the world's richest economies are growing faster than the poorest and that world income distribution is polarized into two groups: rich and poor. In this process of polarization, middle income countries are also divided into rich and poor subgroups. Moreover, these economies sometimes exhibit catching-up and

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overtaking phenomena. This implies that some middle income countries that had been growing steadily get stuck in a low-growth equilibrium while some underdeveloped countries suddenly begin to grow at a faster rate. As a result, an initially poor country sometimes overtakes previously richer ones and the relative GDP ranking of countries changes over time.[1]

This paper is intended to construct a simple endogenous growth model that can explain these phenomena by introducing management of the manufacturing processes, an activity which requires human capital.

Our model is a modification of the standard Romer type R&D-based growth model (Romer 1990) in which R&D is executed for the purpose of obtaining a monopoly profit by innovating a new variety of intermediate goods, and the amount of human resource (labor or human capital) input to the R&D sector is the sole endogenous determinant of the long-run growth rate. In this framework, the present paper specifically addresses the functional differences between human capital and labor in the manufacturing process. For this purpose, we formulate the manufacturing process as a two-stage process that includes management and production. Specifically, labor input efficiency is assumed to be determined through management processes, which require human capital. This setup somewhat resembles those of Tran-Nam, Truong, and Tu (1995) and Goldin and Katz (1998); however, there are some significant differences. Tran-Nam, Truong, and Tu (1995) presume that skilled labor and unskilled labor function as perfect substitutes for adequate skilled labor input, although more unskilled labor is needed for lower amounts of skilled labor input. For this reason, skilled labor is inferred as necessary for production. Goldin and Katz (1998) divide the manufacturing process into machine maintenance and production. They argue that the former is conducted by skilled labor and the latter is undertaken by raw labor.

In contrast to these studies, our model assumes that manufacturing processes are divisible into management and production processes. The management process may include personnel administration, process optimization, labor education, and improving the working environment. In our analysis, raw labor is employed only in production processes, while human capital is applied to manufacturing through managers and to R&D activities through researchers. Human capital is the sole human-resource input into R&D activities. In the following analysis, our model demonstrates that while raw labor and human capital work as perfect substitutes, production without labor is impossible. Thus, in contrast to the Tran-Nam, Truong, and Tu

[1]This phenomenon is sometimes called "leapfrogging" (see e.g., Breiz, Krugman and Tsidon 1993).

(1995) model, our model assumes raw labor to be essential for production. This arrangement enables the specification of human capital as that for final goods production or R&D activities.

The conclusions of this study are summarized as follows: First, our model can mimic the polarization of growth rates. Since the R&D activities are not always profitable, firms do not always conduct them. If R&D activities are profitable, an economy grows with endogenous technological progress; if not, the economy allocates no input for R&D and thus, remains in a no-growth situation. When an economy has scarce resources of labor and human capital, the equilibrium with no R&D input becomes a unique steady state, from which an economy is unable to escape. This is known as a poverty trap, wherein a policy for subsidizing the profits of the intermediate goods sector can be implemented to pull an economy out of the trap. In contrast, the richest countries have greater resources of labor and human capital; therefore, R&D is extremely profitable, since there is a high allocation of human capital for R&D activities. Consequently, these economies grow at a faster rate than others; this results in polarization.

Second, our model can mimic leapfrogging. Countries with median quantities of labor and human capital may have multiple equilibria. If two economies satisfy the conditions for multiple equilibria, we might have a situation in which the country with large R&D investments suddenly switches to the equilibrium with no R&D investment, and the other country with no-R&D investment suddenly jumps to the equilibrium with large R&D investments. Thus, it leapfrogs the other country in terms of growth.

This paper is organized as follows: The next section describes the basic model. Section 3 derives steady state equilibria. Section 4 concludes the paper.

2 The Model

We assume the production structure of Benhabib, Perli, and Xie (1994) which extends the Romer model by allowing complementarities among intermediate inputs. Our model includes three sectors: a final goods sector, an intermediate goods sector, which comprises a continuum of intermediate goods firms indexed by $i \in [0, A]$, and an R&D sector that creates new varieties of goods by employing existing knowledge and human capital. Time is continuous, and a final good is used as a numeraire.

There are four production factors - physical capital (termed as "capital" hereinafter) K, unskilled labor or raw labor ("labor") L, skilled labor or intellectual human capital ("human capital") H, and knowledge A, which is measured in terms of the variety of intermediate goods. Labor, human capital, and capital are direct inputs used in the final goods sector. Knowledge acts as an indirect input in the production of final goods. The intermediate goods firms monopolistically supply intermediate goods by using capital. Monopoly power is assigned by a patent, which an intermediate good firm obtains from an R&D firm. R&D firms produce new varieties of intermediate goods by using human capital and existing knowledge as inputs, and the cost of R&D activities is covered by the return on the sale of the patent. In this study, the human capital H may be used in the final goods sector by managers and in R&D activity by researchers. We assume that human capital can be freely shifted between the final goods production and the R&D sectors. Accordingly, human capital allocation is determined in a manner that equates the wage rates offered by these sectors.

2.1 The Final Goods Sector

It is assumed that final goods are produced by using labor and intermediate goods. The specified production function is defined as

$$Y = (\phi\, L)^{1-\alpha} \left(\int_0^A x(i)^{\frac{\alpha}{\zeta}} di \right)^{\zeta}, \tag{1}$$

where Y, ϕ, L, A, and $x(i)$ are the amount of final goods, labor efficiency, the labor used in producing final goods, the variety index, and the intermediate goods used in sector-i, respectively. $\alpha(0 < \alpha < 1)$ and $1-\alpha$ are the productivity efficiency parameters of intermediate goods and labor, implying that the production function exhibits constant returns to scale. $\zeta(\geq 1)$ captures the degree of complementarity between intermediate goods (if $\zeta = 1$, there is no complementarity). ϕL indicates the labor-augmenting property. The present study assumes that the management process improves the efficiency of raw labor input, ϕ. We specify the efficiency function ϕ as

$$\phi(H_Y, L, A) = A^{\theta} \left(1 + \gamma \frac{H_Y}{L} \right), \quad \gamma > 0, \quad \theta > -\frac{\zeta - \alpha}{1-\alpha}, \tag{2}$$

where γ and θ represent the efficiency derived from applying management ability to labor, and the parameter relating knowledge stock to efficiency of management ability, respectively. Eq. (2) describes the effects of human capital allocated to final goods H_Y, labor L on labor efficiency. Differentiating Eq. (2) with respect to each variable yields the following properties: $\phi_{H_Y} > 0$ and $\phi_L < 0$. $\phi_L < 0$ captures the idea that managers are less effective when the number of workers they supervise increases. The sign of ϕ_A depends on the sign of θ. If $\theta > 0$, then $\phi_A > 0$ and technological progress decreases the necessity of management activities. Further, if $\theta < 0$, then $\phi_A < 0$ and technological progress requires more management activities.

Combining Eqs. (1) and (2) yields the optimizing problem of the final goods firms:

$$\max_{L,H_Y,\{x(i)\}} \quad A^{\theta(1-\alpha)}(L+\gamma H_Y)^{1-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\zeta} - w_L L$$
$$-w_Y H_Y - \int_0^A p(i)x(i)di \quad (\equiv \Pi^Y), \tag{3}$$

where w_L, w_Y, $p(i)$, and Π^Y denote the wage of the laborers, the wage of the human capital employed in the final goods sector, the price of the intermediate goods, and the aggregate profit of the final goods firms, respectively. The firms in the final goods sector are assumed to be price takers that maximize profits. Therefore, they choose the level of input that will equate the marginal production of input factors and the factor price.

The first-order conditions (FOCs) indicate

$$\frac{\partial \Pi^Y}{\partial L} = (1-\alpha)A^{\theta(1-\alpha)}(L+\gamma H_Y)^{-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\zeta} - w_L = 0 \tag{4}$$
$$\frac{\partial \Pi^Y}{\partial H_Y} = \gamma(1-\alpha)A^{\theta(1-\alpha)}(L+\gamma H_Y)^{-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\zeta} - w_Y = 0. \tag{5}$$

Eqs. (4) and (5) indicate that the relationship between the wages of labor and human capital is given by $w_Y = \gamma w_L$.

Our model does not satisfy one of the Inada conditions. From Eq. (5), the upper and lower bounds of the wage, $\bar{w}_Y$ and $\underline{w}_Y$, respectively, are determined on the basis of the condition $H_Y \in [0, H]$ as follows:

$$\bar{w}_Y \equiv w_Y|_{H_Y=0} = \gamma(1-\alpha)A^{\theta(1-\alpha)}L^{-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\zeta} (< \infty),$$
$$\underline{w}_Y \equiv w_Y|_{H_Y=H} = \gamma(1-\alpha)A^{\theta(1-\alpha)}(L+\gamma H)^{-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\zeta} (> 0).$$

Therefore, in this model, w_Y does not satisfy the Inada condition: $\lim_{H_Y \longrightarrow 0} w_Y = \infty$, which implies that human capital is not an essential factor in final goods production.

In order to derive the demand of intermediate goods by the final goods sector, we assume $p(i) = p$, $\forall i$. Subsequently, all intermediate goods are demanded in the same amount: $x(i) = x$, $\forall i$. Using (3), this amount is implicitly given by the FOC:

$$p = \alpha A^{\zeta+\theta(1-\alpha)-1}(L+\gamma H_Y)^{1-\alpha}x^{\alpha-1}. \tag{6}$$

2.2 The Intermediate Goods Sector

An intermediate goods firm i is a firm that possesses a permanent patent on the intermediate goods used in sector i. Consequently, the intermediate goods firm can monopolistically supply the ith intermediate good. It is assumed that one unit of intermediate goods requires η units of physical capital, the rental price of which (that is, the interest rate) is denoted by r. The inverse demand function for an intermediate good x_j is

$$p_j = \alpha A^{\theta(1-\alpha)}(L+\gamma H_Y)^{1-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\zeta-1} x_j^{\frac{\alpha}{\zeta}-1}.$$

Hence, the optimizing problem of an intermediate goods sector is given by

$$\max_{x_j} \quad \alpha A^{\theta(1-\alpha)}(L+\gamma H_Y)^{1-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\zeta-1} x_j^{\frac{\alpha}{\zeta}} - r\eta x_j \quad (\equiv \pi^M).$$

Its solution entails

$$\frac{\alpha^2}{\zeta}A^{\theta(1-\alpha)}(L+\gamma H)^{1-\alpha}\left(\int_0^A x(i)^{\frac{\alpha}{\zeta}}di\right)^{\alpha-1} x_j^{\frac{\alpha}{\zeta}-1} = r\eta.$$

In a symmetric Nash-equilibrium, $x(i) = x$, $\forall i$, which implies

$$\alpha A^{\zeta+\theta(1-\alpha)-1}(L+\gamma H_Y)^{1-\alpha}x^{\alpha-1} = \frac{r\eta\zeta}{\alpha}. \tag{7}$$

From Eqs. (6) and (7), the intermediate goods firms set their prices as $p = \frac{\zeta\eta r}{\alpha}$. This pricing yields the profit of the intermediate goods firms as follows:

$$\pi^M = \frac{\zeta-\alpha}{\alpha}r\eta x. \tag{8}$$

2.3 The R&D Sector

Innovation is assumed to be the discovery of a new design of intermediate goods that are added to the existing set of intermediate goods. R&D firms create the designs of new intermediate goods, and the patents of these designs bear the stream of monopoly profits. The present value of this stream represents the value of R&D:

$$v \equiv \int_0^{\infty} \pi^M(\tau) e^{-\int_0^{\tau} r(s) ds} d\tau.$$

Since the R&D sector is also assumed to be competitive, the value of R&D, v, is equated to the price of a design.

In the process of innovation, it is assumed that R&D firms use human capital and enjoy the free use of knowledge, which is measured by stock of intermediate goods variety. An R&D firm j that employs H_A^j units of human capital produces an output equal to $\delta A H_A^j$ $(\delta > 0)$ and makes a profit

$$\pi^R = v\delta A H_A^j - w_A H_A^j,$$

where w_A is the wage offered in the R&D sector.

Free entry of R&D is assumed. Thus, if $\pi^R > 0$, then an infinite amount of human capital would be allocated to R&D activities; therefore, this cannot hold in equilibrium. On the other hand, if $\pi^R < 0$ holds, then investment in R&D is unprofitable. Consequently, resources are no longer allocated to R&D and an equilibrium without R&D ($H_A = 0$) occurs. However, if $\pi^R = 0$, then a positive amount of human capital would be allocated to R&D and the market would be in equilibrium. This situation arises if

$$v\delta A = w_A \quad \text{or} \quad V \equiv vA = \frac{w_A}{\delta}, \tag{9}$$

where $V \equiv vA$ is the total production of the R&D sector. If $V < w_A/\delta$, there will be no R&D. If R&D is undertaken, technological knowledge imposes according to

$$\frac{\dot{A}}{A} = \delta H_A. \tag{10}$$

2.4 Key Dynamic Equations

This subsection introduces some important dynamic equations. Since capital is used only in the intermediate goods sector and η units of intermediate goods are produced from one unit of physical capital, the aggregate level of physical capital K is defined as

$$K \equiv \int_0^A \eta\, x(i) di = \eta A\, x. \tag{11}$$

Since H is used either in final goods production or in the R&D sector, the ratio of human capital devoted to the management process, H_Y, to the aggregate human capital, H, can be written as $s \equiv \frac{H_Y}{H}$. According to this definition, $1-s=\frac{H_A}{H}$ represents the ratio of human capital devoted to R&D, H_A, to the aggregate human capital, H. Under this expression, the amount of human resource allocated to final goods production is defined as

$$N(s) \equiv L + \gamma s H = L + \gamma H_Y.$$

The definition of $N(s)$ dictates that $N(1) = L + \gamma H \equiv N$ corresponds to the sum of manual labor and effective intellectual human capital, i.e., the total human resources. It is noteworthy that this human resource index N is associated with the manufacturing of final goods through the efficiency function ϕ in the final goods production.

Eliminating x from Eq. (1) by using the definition of Eqs. (2) and (11), and incorporating the expression $N(s)$, we derive the aggregate product

$$Y = \eta^{-\alpha} A^{\psi} N(s)^{1-\alpha} K^{\alpha}, \tag{12}$$

where $\psi \equiv \zeta - \alpha + \theta(1-\alpha) > 0$. From Eqs. (7), (11), and (12), the interest rate is given as

$$r = \frac{\alpha^2}{\zeta}\frac{Y}{K}. \tag{13}$$

Using (5), (8) and the variable Y, the profit of the intermediate goods sector and the wage determined by the final goods sector can be rewritten as follows:

$$\pi^M = \frac{\alpha(\zeta-\alpha)}{\zeta}\frac{Y}{A} \qquad \text{and} \qquad w_Y = \gamma(1-\alpha)\frac{Y}{N(s)}. \tag{14}$$

In equilibrium, the arbitrage condition with respect to wage rates equates w_A given by Eq. (9) and w_Y in Eq. (14) through the allocation of human capital. However, if one wage rate is higher than the other at the boundary of allocation $s = 1$ or $s = 0$, all human capital will be employed in the sector that offers the higher wage rate. If $w_Y > w_A$, then $s = 1$, and if $w_Y < w_A$, then $s = 0$. These conditions are summarized as follows:

$$\left.\begin{array}{l} s = 0 \\ s\text{: inner solution} \\ s = 1 \end{array}\right\} \Longleftrightarrow w_A \left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} w_Y. \tag{15}$$

Then the wage of human capital w_H is determined as $w_H = \max\{w_A, w_Y\}$. The next section presents a detailed analysis of this condition.

There exist two types of households in our economy: households with (raw) labor and households with human capital. Consequently, there are two optimizing problems. However, since the instantaneous utility function is assumed to be a constant relative risk aversion (CRRA) type, we convert the two optimizing problems to one aggregative problem[2], as

$$\begin{aligned} \max \quad U_t &= \int_0^\infty \frac{C^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt, \quad \rho > 0, \quad \sigma > 0, && (16) \\ \text{s.t.} \quad \dot{K} &= rK + w_L L + w_H H + \Pi^M - C, && (17) \end{aligned}$$

where ρ and σ are the discount rate and CRRA parameters, respectively.

By solving the above problem using this instantaneous utility function, the Keynes-Ramsey rule is obtained as

$$\sigma \frac{\dot{C}}{C} + \rho = r. \tag{18}$$

The transversality condition (TVC) is given as

$$\lim_{t \to \infty} e^{-\rho t} \lambda_t K_t = 0, \tag{19}$$

where $\lambda (\equiv C^{-\sigma})$ is a shadow price of capital stock K.

A market-clearing condition is obtained by substituting N, w_Y, and Eqs. (13) and (14) into Eq. (17); this yields the following equation:

$$\dot{K} = Y - C. \tag{20}$$

Differentiating V with respect to time, we have the following dynamic equation:

$$rV = \dot{V} + \Pi^M, \tag{21}$$

where Π^M denotes the aggregate profit of the intermediate goods sector defined as $\Pi^M \equiv \int_0^A \pi(j)^M dj$, which is calculated from Eq. (14) as

$$\Pi^M = A\pi^M = \frac{\alpha(\zeta - \alpha)}{\zeta} Y. \tag{22}$$

The economy is endowed with a fixed stock of unskilled labor and human capital at every instant of time, that is,

$$\dot{L} = \dot{H} = 0.$$

These dynamic equations constitute the system in this model.

The model contains two state variables whose initial values are historically given as physical capital K and technological knowledge stock A. Therefore,

[2] Since this utility function is homothetic, the reduced consumption functions with respect to wealth are linear. Thus, these two functions can be aggregated.

it displays transitional dynamics; however, as shown later, the model has four types of steady states, which always contain at least one saddle stable path under some restrictions[3]. Therefore, we will now concentrate our attention on the steady state analysis.

3 Steady State

3.1 Variables in Steady States

A steady state implies that each variable grows at a constant rate. The growth rate of variable Z is written as g_Z in this paper.

The dynamic equation of patent accumulation (10) implies that the growth rate of knowledge is

$$g_A = \delta H_A = \delta(1-s)H\,. \tag{23}$$

From this equation, s is constant in the steady state. From Eq. (20),

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K}$$

holds. Accordingly, in the steady state, $g_K = g_Y = g_C (\equiv g^*)$ is necessary for g_K to be constant. From Eq. (12), the relationship among the growth rates of Y, K, and A is given as

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + \psi\frac{\dot{A}}{A}.$$

The above equation and $g_K = g_Y = g^*$ demonstrate the following relationship between the growth rate of physical capital and knowledge:

$$g^* = \frac{\psi}{1-\alpha} g_A^* = \frac{\psi}{1-\alpha}\delta(1-s^*)H, \tag{24}$$

where s^* is the value of s in a steady state.

It is useful to express the system in terms of variables that will be constant in a steady state. From Eq. (24), the knowledge-adjusted variable $\hat{X}$ for variable X can be defined as follows:

$$\hat{X} \equiv \frac{X}{A^{\frac{\psi}{1-\alpha}}}\,, \quad \text{for} \quad X = Y, K, \text{ and } C. \tag{25}$$

[3]The stability analysis of the current model is offered in the longer version of this paper, that can be downloaded at http://www.geocities.jp/kuwahala/MA.pdf

By this expression, Eqs. (12) and (13) can be rewritten as

$$\hat{Y} = \eta^{-\alpha} N(s)^{1-\alpha} \hat{K}^{\alpha}, \quad \text{and} \quad r = \frac{\alpha^2}{\zeta}\frac{\hat{Y}}{\hat{K}}. \tag{26}$$

These variables are constant in the steady state. From (19) and (24), the TVC is rewritten as

$$\rho > (1-\sigma)g(s^*). \tag{27}$$

3.2 The R&D Condition

This subsection presents the condition for positive R&D activities in the steady state. Eq. (21) is rewritten as

$$r - g_V = \frac{\Pi^M}{V}. \tag{28}$$

Since r and g_V must be constant in the steady state, Eq. (28) implies that $g_V = g_{\Pi^M}$ holds in the steady state. From Eq. (22), $g_{\Pi^M} = g_Y = g^*$ is necessary in the steady state. Combining $g_V = g_{\Pi^M}$ and $g_{\Pi^M} = g_Y = g^*$, we have $g_V = g^*$ in the steady state. Substituting $g_V = g^*$ and $w_A = \delta V$ from Eq. (9) and Π^M from Eq. (22) into Eq. (28), and solving it with respect to w_A, we obtain the following equation:

$$w_A = \frac{\delta\alpha(\zeta-\alpha)Y}{\zeta(r-g^*)}. \tag{29}$$

According to TVC and Keynes-Ramsey rule, $r - g^* = \rho - (1-\sigma)g^* > 0$, which yields $w_H > 0$.

Substituting Eqs. (14) and (29) into Eq. (15) yields the following relationships between s and wages:

$$\left.\begin{array}{l} s^*=0 \\ s^* \in (0,1) \\ s^*=1 \end{array}\right\} \Longleftrightarrow w_A = \frac{\alpha\delta(\zeta-\alpha)Y}{\zeta(r-\frac{\psi}{1-\alpha}g_A)} \left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} \gamma(1-\alpha)\frac{Y}{N(s^*)} = w_Y.$$

Eliminating Y and r by using Eqs. (24) and (26), the above arbitrage condition is solved with respect to $\hat{K}$ as

$$\hat{K}\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\}\left[\frac{\frac{\alpha^2}{\zeta}\eta^{-\alpha}}{\frac{\delta(\zeta-\alpha)}{\gamma\zeta(1-\alpha)}\alpha N(s^*) + \frac{\psi}{1-\alpha}\delta(1-s^*)H}\right]^{\frac{1}{1-\alpha}} N(s^*) (\equiv \hat{K}(s^*)^{RD}).$$

$$\Longleftrightarrow \left\{\begin{array}{l} s^*=0 \\ s^* \in (0,1) \\ s^*=1 \end{array}\right. \tag{RD}$$

The R&D condition (RD) has the following properties: When $s^* \in (0,1)$, there exists a corresponding equilibrium knowledge-adjusted capital stock $\hat{K} = \hat{K}(s^*)^{RD}$. Along the lines of $s = 0$ and $s = 1$, $\hat{K} \geq \hat{K}(0)^{RD}$ and $\hat{K} \leq \hat{K}(1)^{RD}$ satisfy condition (RD), respectively[4]. Therefore, the set $(s^*, \hat{K})$, satisfying condition (RD), is drawn as an R&D line in Figure 1.

3.3 The Keynes-Ramsey Line

This subsection derives the equilibrium condition, which stems from the optimization of households. In a steady state, the knowledge-adjusted consumption level must be constant:

$$\frac{\dot{\hat{C}}}{\hat{C}} = \frac{\dot{C}}{C} - \frac{\psi}{1-\alpha}\delta(1-s)H = 0\,.$$

Substituting the interest rate given by Eq. (26) and the Keynes-Ramsey rule (18) into this equation, the steady-state knowledge-adjusted capital stock must satisfy the following equation:

$$\frac{1}{\sigma}\left(\frac{\alpha^2}{\zeta}\eta^{-\alpha}N(s^*)^{1-\alpha}\hat{K}^{\alpha-1} - \rho\right) = \frac{\psi}{1-\alpha}\delta(1-s^*)H.$$

This equation yields the knowledge-adjusted capital stock in a steady state as a function of s^*:

$$\hat{K} = \left[\frac{\frac{\alpha^2}{\zeta}\eta^{-\alpha}}{\sigma\frac{\psi}{1-\alpha}\delta(1-s^*)H + \rho}\right]^{\frac{1}{1-\alpha}} N(s^*)(\equiv \hat{K}(s^*)^{KR}). \qquad (KR)$$

This equation is derived from the household's optimal condition, the Keynes-Ramsey rule. For this reason, we refer to this equation as the Keynes-Ramsey line. Appendix A.2 shows that (KR) is an increasing function on the s^*-$\hat{K}$ plane. The line is drawn in each panel of Figure 1 as a KR-line.

3.4 Steady States

3.4.1 Three Types of Equilibria

In this subsection, we derive the equilibria of human capital allocation. There are three types of equilibria: a "no-R&D equilibrium" ($s^* = 1$), in which

[4]The equilibria with $s^* = 0$ and $s^* = 1$ are the specialized equilibria pertaining to the human capital allocation between the R&D sector and the final goods sector. They exist because both wage rates do not diverge when the human capital input for each sector tends to be 0.

all human capital is devoted to the manufacture of final goods, a "R&D-Specialized Equilibrium" ($s^* = 0$), in which all human capital is devoted to R&D activities, and a "Diversified Equilibrium" ($s^* = \bar{s} \in (0,1)$), in which human capital is the input for both final goods and R&D sectors (See Appendix A.2 for detailed derivations.).

No-R&D Equilibrium (NE) When $s^* = 1$ holds in equilibrium, a no-R&D equilibrium (NE) emerges. The condition for this equilibrium is presented as $\hat{K}(1)^{KR} < \hat{K}(1)^{RD}$. It is summarized as the following inequality:

$$N < B(\alpha, \gamma, \delta, \rho, \zeta) \left(\equiv \frac{\gamma\rho\zeta(1-\alpha)}{\alpha\delta(\zeta-\alpha)} \right). \tag{PT}$$

Equation (PT) shows that poverty traps emerge when the available human resources are smaller than the threshold $B(\alpha, \delta, \rho, \zeta)$. Since $B_\alpha < 0$, $B_\delta < 0$, $B_\rho > 0$, and $B_\zeta < 0$, a higher level of human resources N are required to escape from the poverty trap in cases where (i) the R&D efficiency δ is low, (ii) the efficiency of intermediate goods on final goods production α is low, (iii) the subjective discount rate ρ is high, or and (iv) the complementarity between intermediate goods is low. Since (PT) is transformed into $L = \gamma(\tilde{B} - H)$, where $\tilde{B} = B/\gamma = \frac{\rho\zeta(1-\alpha)}{\alpha\delta(\zeta-\alpha)}$, the large population is necessary for the higer effects of human capital management ability and low human capital endowment.

R&D-specialized Equilibrium (RE) In the case of $s^* = 0$, all human capital is devoted to R&D activities. The condition is given as $\hat{K}(0)^{KR} > \hat{K}(0)^{RD}$, and can be made into the following inequality:

$$\tilde{N} > B(\alpha, \gamma, \delta, \rho, \zeta), \tag{RS}$$

where $\tilde{N} \equiv L + \frac{\gamma\zeta\psi(1-\sigma)}{\alpha(\zeta-\alpha)} H$, and $\tilde{N}$ represents the threshold level of human resources required to create an RE.

Diversified Equilibrium (DE) When the share of human capital is written as $\bar{s}(\in (0,1))$, then the human capital is diversified into both R&D and final goods production sectors, which is referred to as DE. A DE could emerge when the following conditions are satisfied

$$N < B(\alpha, \gamma, \delta, \rho, \zeta) < \tilde{N} \quad \text{or} \quad \tilde{N} < B(\alpha, \gamma, \delta, \rho, \zeta) < N.$$

As discussed later, the former condition relates to cases of multiple equilibria, where $\bar{s}$ is one of the multiple equilibria and is associated with positive growth. The latter represents the condition for a unique equilibrium with positive growth.

3.4.2 Types of Steady States

In this subsection, four types of steady states are derived by investigating the conditions of the equilibria. All these types are shown in Figure 1; the relationship between steady states and equilibria is given in Table 1.

Case A. Growth with R&D-specialized Human Capital (RS) Case A has only one equilibrium with $s^* = 0$.

$$\hat{K}(0)^{KR} > \hat{K}(0)^{RD} \quad \text{and} \quad \hat{K}(1)^{KR} > \hat{K}(1)^{RD}.$$

This case is depicted in Figure 1(a). The steady state is uniquely determined, and all human capital is devoted to the R&D sectors. From the above conditions, we derive the following: $\min(N, \tilde{N}) > B$. This condition implies that both the human resource indices are larger than the threshold B.

Case B. Growth with Diverged Human Capital (D) Case B has a unique inner solution $s^* = \bar{s}$.

$$\hat{K}(0)^{KR} < \hat{K}(0)^{RD} \quad \text{and} \quad \hat{K}(1)^{KR} > \hat{K}(1)^{RD}.$$

The situation is depicted in Figure 1(b). An inner solution of s^* is uniquely determined. The economy grows endogenously. When (PT) and (RS) do not hold simultaneously, only one inner solution exists. This situation is characterized by the condition $\tilde{N} < B < N$, which indicates that human resources N are large, but R&D-related human resources $\tilde{N}$ are sufficiently small. The condition in this case implies $N > \tilde{N}$, which requires

$$\sigma > \bar{\sigma} \left(\equiv 1 - \frac{\alpha(\zeta - \alpha)}{\zeta \psi(\theta)} \right).$$

Since $\bar{\sigma}_\zeta < 0$, $\bar{\sigma}_\psi > 0$, and $\psi_\theta > 0$, large σ and ζ, and a small θ are necessary for satisfying this condition.

Case C. No Growth or Poverty Traps (NG) A "poverty trap" is defined as a situation wherein the economy stagnates. The case mentioned in this paper has a unique NE ($s^* = 1$) and its conditions are given as follows:

$$\hat{K}(0)^{KR} < \hat{K}(0)^{RD} \quad \text{and} \quad \hat{K}(1)^{KR} < \hat{K}(1)^{RD}.$$

This case is depicted as Figure 1(c). These conditions can be summarized as $\max(N, \tilde{N}) < B$. This implies that both the human resource indices are smaller than the threshold B.[5]

[5] This steady state can be eliminated by subsidizing R&D profits. Consider that a constant rate tax $\tau > 0$ (a subsidy of $\tau < 0$) is provided for the interest (the rental price

Case D. Multiple Equilibrium (ME) This case contains three equilibria, $s^* = 0, \bar{s}, 1$.

$$\text{Fig.1(d)} \Longleftrightarrow \hat{K}(0)^{KR} > \hat{K}(0)^{RD} \quad \text{and} \quad \hat{K}(1)^{KR} < \hat{K}(1)^{RD}.$$

When (PT) and (RS) hold simultaneously, both points of $s^* = 1$ and $s^* = 0$ are equilibria (and furthermore, there exists an inner solution). This case is depicted in Figure 1(d). This yields the condition $N < B < \tilde{N}$: human resources N are small, but R&D-related human resources $\tilde{N}$ are adequate. The condition $\tilde{N} > N$ implies that $\sigma < \bar{\sigma}$. Figure 1(d) depicts this case. Because $\psi_\theta > 0$, small γ and σ, and large θ and ζ are necessary for the multiple steady states.

From cases B and D, we obtain the result that a small CRRA parameter and a middle range of human resource endowment are necessary for multiple equilibria. Stated differently, countries with median human resources can achieve leapfrogging or decomposition.

4 Conclusions

This study establishes an R&D-based growth model that includes the functional difference between labor and human capital in the production of goods. Our model assumes that the manufacturing processes are divisible into the management and production processes and that human capital is used in the management process, while labor is used in the production process. This arrangement yields one of the following steady states: long-run positive R&D-based growth, poverty traps, and multiple equilibria. Therefore, the steady states provided by this study are consistent with large diversities in economic growth rates, which result in polarization and leapfrogging. The results obtained and the mechanisms adopted are summarized as follows:

First, a small endowment of human resources implies less demand for innovation resulting in high costs of R&D activities. Since the range of the wage rate, which can vary through the allocation of human capital between

of capital) and the profit of the intermediate sector: $r^\tau = (1-\tau_r)r$ and $\pi^\tau = (1-\tau_\pi)\pi^M$, where τ_r and τ_π are the interest tax and the profit tax, respectively. Translating r and π in (KR) and (RD) to r^τ and π^τ, we can obtain the KR and RD lines after being taxed (or subsidized). Human capital allocation in the steady state depends entirely on the positional relationship between the KR and RD lines. We can easily observe that the change in the interest tax has no effect on this positional relationship since any change in τ_r produces merely a parallel shift of both the lines. As a result, equilibrium capital allocation s^* does not change at all; therefore, the change in interest tax is not an effective economic policy for long-run growth. In contrast, τ_π only effects the RD line. If $\tau_\pi < 0$, then the RD line shifts upward, thus, eliminating the NG equilibrium.

the two sectors, is restricted, all human capital may be concentrated toward the manufacturing of final goods. Thus, no R&D activity is undertaken. In this situation, the economy lacks technological progress and falls into a poverty trap. This equilibrium can be eliminated by enacting certain policies, for example, subsidizing the profits of the intermediate goods firms. In a case where a country has sufficient human resources to ensure that R&D is profitable, R&D is conducted constantly and the economy grows continuously. This results in the polarization of economies. Second, when economies with median endowments of human resources have a higher parameter of intertemporal elasticity of substitution, ME may occur. The case of ME contains two saddle stable paths converging to NE and RE. Therefore, growth rates can change depending on the equilibrium R&D investment and may generate leapfrogging.

Appendix

A.1 Stability Analysis

Corresponding to the allocation of human capital between R&D and management, the model of this study has three types of steady states:

$$\begin{array}{rlll} \text{Diversified Equilibrium (DE)} & s^* = \bar{s} \in (0,1), & \text{or} & N(\bar{s}) = L + \gamma \bar{s} H, \\ \text{No-R\&D Equilibrium (NE)} & s^* = 1, & \text{or} & N(1) = L + \gamma H, \\ \text{R\&D-Specialized Equilibrium (RE)} & s^* = 0, & \text{or} & N(0) = L. \end{array}$$

The index of DE, NE, and RE are given as $*1$, $*2$, and $*3$, respectively.

The stability of the model is given as follows:

Let $\chi \equiv C/K$, and substituting χ and $r = (\alpha^2/\zeta)(Y/K)$ (from (13)) into $\dot{K}/K = Y/K - C/K$ (from (20)), we obtain

$$g_K = \frac{\zeta}{\alpha^2} r - \chi, \tag{30}$$

where $g_Z \equiv \dot{Z}/Z$. We define $n \equiv N(s) = L + \gamma s H$, and because $s \in [0,1]$, $n \in (L, N)$.

Equation (10) and the definition of n, knowledge growth is governed by

$$g_A = \frac{\delta}{\gamma}(N - n). \tag{31}$$

From Eqs. (18) and (30), the dynamics of χ are given as

$$g_\chi = \left(\frac{1}{\sigma} - \frac{\zeta}{\alpha^2}\right) r - \frac{\rho}{\sigma} + \chi. \tag{A1}$$

When $n \in (L, N)$ hold, $V = w_Y/\delta$ must be satisfied. Differentiating $V = w_Y/\delta$, $w_Y = \gamma(1-\alpha)Y/n$, and $r = (\alpha^2/\zeta)(Y/K)$ with respect to time yield $g_V = g_{w_Y} = g_Y - g_n = g_r + g_K - g_n$. Substituting this, (30) and

$$\Pi^M = \frac{\alpha(\zeta - \alpha)}{\zeta} Y$$

into $r = g_V + \Pi^M/V$, we obtain

$$g_n = g_r + \frac{\zeta}{\alpha^2} r - \chi - r + \frac{\alpha\delta(\zeta-\alpha)}{\gamma\zeta(1-\alpha)} n. \tag{32}$$

Substituting $g_r = g_Y - g_K$ (from (13)), (30), and (31) into $g_Y = \psi g_A + (1-\alpha)g_n + \alpha g_K$ (from (12)), we derive

$$g_r = \frac{\delta}{\gamma}\psi(\theta)(N-n) + (1-\alpha)g_n + (\alpha - 1)\left(\frac{\zeta}{\alpha^2} r - \chi\right). \tag{33}$$

Eliminating $g_n - (\zeta/\alpha^2)r + \chi$ by using (32) and (33) provides the dynamics of r as follows:

$$g_r = \frac{\delta}{\alpha\gamma}\psi(\theta)(N-n) - \frac{1-\alpha}{\alpha} r + \frac{\zeta - \alpha}{\gamma\zeta}\delta n. \tag{A2}$$

Substituting (A2) into (32), we obtain

$$g_n = \frac{\delta}{\alpha\gamma}\psi(\theta)(N-n) + \frac{\delta(\zeta-\alpha)}{\gamma\zeta(1-\alpha)} n + \left(\frac{\zeta}{\alpha^2} - \frac{1}{\alpha}\right) r - \chi. \tag{A3}$$

(A1), (A2), and (A3) depict the dynamics of the model of this study.

The case of DE Using (A1), $g_\chi = 0$, and $\frac{\psi(\theta)}{1-\alpha} g_A = \frac{1}{\sigma}(r - \rho)$, the value of χ in a steady state can be given as

$$\chi^* = -\frac{\psi(\theta)}{1-\alpha} g_A^* + \frac{\zeta}{\alpha^2} r^*. \tag{34}$$

From $g_r = 0$, we immediately obtain the value of r in a steady state as

$$r^* = \frac{\psi(\theta)}{1-\alpha} g_A^* + \frac{\alpha\delta(\zeta-\alpha)}{\gamma\zeta(1-\alpha)} n^*. \tag{35}$$

By substituting $r - \rho = \sigma \frac{\psi(\theta)}{1-\alpha} g_A$ into (35), n^{*1} is derived as

$$n^{*1} = \frac{(1-\alpha)\gamma\rho - (1-\sigma)\delta\psi(\theta)N}{(\alpha\delta)(\zeta-\alpha) - (1-\sigma)\delta\psi(\theta)}.$$

The Jacobian in this case is given as

$$J^{*1} \equiv \begin{pmatrix} \chi^{*1} & \left(\frac{1}{\sigma} - \frac{\zeta}{\alpha^2}\right)\chi^* & 0 \\ 0 & -\left(\frac{1}{\alpha} - 1\right) r^* & \left(-\frac{\delta}{\alpha\gamma}\psi(\theta) + \frac{\delta(\zeta-\alpha)}{\gamma\zeta}\right) r^* \\ -n^* & \frac{\zeta-\alpha}{\alpha^2} n^* & \left(-\frac{\delta}{\alpha\gamma}\psi(\theta) + \frac{\delta(\zeta-\alpha)}{\gamma\zeta(1-\alpha)}\right) n^* \end{pmatrix}.$$

By using (34) and (35), the trace of J^{*1} is calculated as

$$Tr^{*1} = \underbrace{\frac{\zeta-\alpha}{\alpha^2} r^*}_{\text{positive}} + \underbrace{\left[\frac{\delta(\zeta-\alpha)(1+\alpha)}{\gamma\zeta(1-\alpha)} - \frac{\delta}{\alpha\gamma}\psi(\theta)\right] n^*}_{\text{positive or negative}}.$$

This equation implies that if $\psi(\theta) < \frac{(\zeta-\alpha)(1+\alpha)}{\zeta(1-\alpha)}\alpha$, then $Tr^{*1} > 0$. For example, a small θ realizes $Tr^{*1} > 0$.

The determinant of J^{*1} is given as

$$Det^{*1}(\chi^{*1}, r^{*1}, n^{*1}) = -\left[\frac{1}{\sigma}\left(\frac{\delta(\zeta-\alpha)}{\gamma\zeta} - \frac{\delta}{\alpha\gamma}\psi(\theta)\right) + \frac{\delta\psi(\theta)}{\alpha\gamma}\right] \chi^{*1} r^{*1} n^{*1}.$$

Since χ^{*1}, r^{*1}, and n^{*1} are positive, the condition $Det^{*1} < 0$ requires

$$\frac{1}{\sigma}\left(\frac{\delta(\zeta-\alpha)}{\gamma\zeta} - \frac{\delta}{\alpha\gamma}\psi(\theta)\right) + \frac{\delta}{\alpha\gamma}\psi(\theta) > 0.$$

Solving this condition with respect to σ, we obtain

$$\sigma > 1 - \frac{\alpha(\zeta-\alpha)}{\zeta\psi(\theta)} (= \bar{\sigma})$$

It should be denoted that this condition is the same one in which the case of Growth with Diversify Equilibria (D) emerges. Therefore, a combination of sufficiently small $\psi(\theta)$ and σ yield $Det^{*1} < 0 < Tr^{*1}$, which guarantees that the economy has a unique saddle-stable path converging to the steady state $\{\chi^{*1}, r^{*1}, n^{*1}\}$. The case of ME also contains equilibrium $\{\chi^{*1}, r^{*1}, n^{*1}\}$; however, the equilibrium does not have a saddle stable path.

The case of NE In this case, all human capital is employed on the management in the final goods sector - $n = N$ holds. $N < B(\alpha, \delta, \rho, \zeta)$ is necessary for this equilibrium to exist. In equilibrium, $n^* = N$ yields $g_Y = g_K = g_C = g_A = 0$. Accordingly, Eq. (18) provides $r^* = \rho$, and this $r^* = \rho$ and Eq. (20) give $\chi^* = Y/K = (\zeta/\alpha^2)r^* = (\zeta/\alpha^2)\rho$. Since $n = N$ is a corner solution,

$\dot{n}|_{n=N}^{*2} > 0$ is necessary for the convergence from $n < N$ to $n = N$. However, substituting $\{\chi^{*2}, r^{*2}, n^{*2}\} = \{(\zeta/\alpha^2)\rho, \rho, N\}$ into (A3) yields

$$g_n|_{n=N}^{*2} = \frac{\delta(\zeta-\alpha)}{\zeta(1-\alpha)}(N - B(\alpha,\delta,\rho,\zeta)) < 0.$$

Therefore, the path converging to NE must satisfy $n = N$ in the neighborhood of NE. In this case, it is convenient to describe the system by using $\hat{K}$, $\hat{C}$, and $\tilde{v}(\equiv V/Y)$. Since the dynamics of $\hat{K}$ and $\hat{C}$ do not include $\tilde{v}$, and the dynamics of $\tilde{v}$ depend on $\hat{K}$ and $\tilde{v}$, the dynamic system is separable into two two-dimensional systems, $\{\hat{K}, \hat{C}\}$ and $\{\hat{K}, \tilde{v}\}$. We first consider the former system, $\{\hat{K}, \hat{C}\}$, following which we consider the latter, $\{\hat{K}, \tilde{v}\}$.

In this case $n = N$; therefore, $g_A = 0$. The dynamics of $\hat{K}$ and $\hat{C}$ are depicted as

$$\begin{aligned}\dot{\hat{K}} &= \eta^{-\alpha}\hat{K}^{\alpha} - \hat{C},\\ \dot{\hat{C}} &= \frac{1}{\sigma}\left(\frac{\alpha^2}{\zeta}\eta^{-\alpha}\hat{K}^{\alpha-1} - \rho\right)\hat{C}.\end{aligned}$$

This is the same form of the simple Ramsey-Cass-Koopmans model; therefore, $\hat{K}$ and $\hat{C}$ have a unique saddle-stable path converging to the steady state $(\hat{K}^{*2}, \hat{C}^{*2})$.

From (21), the dynamics of $\tilde{v}$ can be rewritten as

$$\dot{\tilde{v}} = -\underbrace{\frac{\alpha^2}{\zeta}\eta^{-\alpha}\hat{K}^{\alpha-1}}_{r}\tilde{v} + \underbrace{\frac{\alpha(\zeta-\alpha)}{\zeta(1-\alpha)}}_{\Pi^M/Y}.$$

Therefore, the dynamics of $\tilde{v}$ are governed by

$$\dot{\tilde{v}}\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} 0 \Longleftrightarrow \tilde{v}\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\}\frac{(\zeta-\alpha)\eta^{\alpha}}{\alpha(1-\alpha)}\hat{K}^{1-\alpha}.$$

Investigating this condition and the converging property of $\hat{K}$, we obtain the phase diagram in Figure 2(a).

An unprofitable condition of R&D must be satisfied on the path and steady states. The condition is calculated as $\tilde{v} < \gamma(1-\alpha)/(\delta N)$ from

$$V < w_Y/\delta.$$

The threshold line of the condition is given as line $w_Y/(\delta Y) = \gamma(1-\alpha)/(\delta N)$.

The case of RE This case in which $n = L$ holds implies that all human capital is employed in R&D activities. $\tilde{N} > B(\alpha, \delta, \rho, \zeta)$ is necessary for the existence of this equilibrium. In equilibrium, $n^* = L$ yields $g_Y = g_K = g_C = \frac{\psi}{1-\alpha} g_A = \frac{\psi}{1-\alpha}\delta H$. Accordingly, Eq. (18) provides $r^* = \rho + \frac{\psi\sigma}{1-\alpha}\delta H$, and this condition and Eq. (20) result in $\chi^* = (\zeta/\alpha^2)r^* + \frac{\psi\sigma}{1-\alpha}\delta H$. Since $n = L$ is a corner solution, $\dot{n}|^{*3}_{n=L} < 0$ is necessary for the convergence from $n > L$ to $n = L$. However, substituting $\{\chi^{*3}, r^{*3}, L\}$ into (A3) yields

$$g_n|^{*3}_{n=L} = \frac{\delta(\zeta-\alpha)}{\zeta(1-\alpha)}(\tilde{N} - B(\alpha, \gamma, \delta, \rho, \zeta)) > 0$$

Therefore, the path converging to RE must satisfy $n = L$ in the neighborhood of NE. In this case, it is convenient to describe the system by using $\hat{K}$, $\hat{C}$, and $\tilde{v}$, and the system in this case is also separable into two two-dimensional systems $\{\hat{K}, \hat{C}\}$ and $\{\hat{K}, \tilde{v}\}$.

Regarding $\hat{K}$ and $\hat{C}$, the dynamical system is depicted as

$$\begin{aligned} \dot{\hat{K}} &= \eta^{-\alpha}\hat{K}^{\alpha} - \hat{C} - g_A^{*3}\hat{K}, \\ \dot{\hat{C}} &= \left\{ \frac{1}{\sigma}\left(\frac{\alpha^2}{\zeta}\eta^{-\alpha}\hat{K}^{\alpha-1} - \rho \right) - g_A^{*3} \right\} \hat{C}, \end{aligned}$$

where $g_A^{*3} = \frac{\psi(\theta)\delta}{1-\alpha}H$. This is the same form of the Ramsey-Cass-Koopmans model with a constant rate of exogenous technological progress, therefore, $\hat{K}$ and $\hat{C}$ have a unique saddle-stable path converging to the steady state $(\hat{K}^{*3}, \hat{C}^{*3})$.

In this case, we can demonstrate the same dynamical properties as with the case of NE. Subsequently, we have the phase diagram in Figure 2(b). In this case, $w_A > w_Y$ must be satisfied on the path and the steady state. This condition is given as line $w_Y/(\delta Y) = \gamma(1-\alpha)/(\delta L)$.

A.2 Equilibrium Human Capital Allocation

In the steady state, the human capital allocation is determined by conditions (RD) and (KR).

The R&D condition determines the equilibrium human capital allocation derived from the arbitrage of wage, which is summarized as the following correspondence:

$$\hat{K} \begin{cases} > \hat{K}(0)^{RD}, & \text{if } s^*{=}0, \\ = \hat{K}(s^*)^{RD}, & \text{if } s^* \in (0,1), \\ < \hat{K}(1)^{RD}, & \text{if } s^*{=}1, \end{cases} \qquad (RD)$$

where $\hat{K}(s^*)^{RD}$ is derived as

$$\hat{K} = \left[\frac{\frac{\alpha^2 \eta^{-\alpha}}{\zeta}}{\frac{\alpha\delta(\zeta-\alpha)}{\gamma\zeta(1-\alpha)} N(s^*) + \frac{\psi}{1-\alpha}\delta(1-s^*)H)} \right]^{\frac{1}{1-\alpha}} N(s^*)(\equiv \hat{K}(s)^{RD}).$$

The household's optimization for consumption derives the Keynes-Ramsey Rule, which is written as

$$\hat{K}^{KR} = \left[\frac{\frac{\alpha^2}{\zeta}\eta^{-\alpha}}{\frac{\psi\delta\sigma}{1-\alpha}(1-s^*)H + \rho} \right]^{\frac{1}{1-\alpha}} N(s^*)(\equiv \tilde{K}(s^*)^{KR\frac{1}{1-\alpha}} N(s^*)). \qquad (KR)$$

The derivative of $\tilde{K}(s^*)^{KR}$ is calculated as $\frac{d\tilde{K}(s^*)^{KR}}{ds} > 0$. From this, we infer that the KR line is increasing with s^* because

$$\frac{d\tilde{K}(s^*)^{RD}}{ds^*} = \frac{1}{1-\alpha} \tilde{K}^{KR\frac{\alpha}{1-\alpha}} \frac{d\tilde{K}(s^*)^{KR}}{ds} N(s^*) + \tilde{K}^{KR\frac{1}{1-\alpha}} \gamma H > 0.$$

We can obtain the analytical solutions $(s^*, \hat{K})$ that satisfy (RD) and (KR). There are three types of solutions. One is a case of NE with $s^* = 1$. The other two cases are of positive growth ($s^* = \bar{s} \in [0,1)$): one is the case with a corner solution ($s^* = 0$), termed the RE; the other has an inner solution ($s^* = \bar{s} \in (0,1)$), termed DE. Thus NE, RE, and DE are derived by the conditions $\hat{K}(1)^{KR} < \hat{K}(1)^{RD}$, $\hat{K}(0)^{KR} > \hat{K}(0)^{RD}$, and $\bar{s} = \arg_{s^*}\{\tilde{K}(s^*)^{RD} = \tilde{K}(s^*)^{KR}\}$, respectively.

Substituting (KR) and (RD) into conditions $\hat{K}(1)^{KR} < \hat{K}(1)^{RD}$ and $\hat{K}(0)^{KR} > \hat{K}(0)^{RD}$, respectively, we obtain the following conditions

$$N < B(\alpha, \gamma, \delta, \rho, \zeta) \equiv \frac{\gamma\rho\zeta(1-\alpha)}{\alpha\delta(\zeta-\alpha)} \qquad (PT)$$

and

$$\tilde{N} \equiv L + \frac{\gamma\zeta\psi}{\alpha(\zeta-\alpha)}(1-\sigma)H > B(\alpha, \gamma, \delta, \rho, \zeta), \qquad (RS),$$

respectively. The first equation is the condition for the NE; the second equation is the condition for RE.

Section 2.3 defined N as human resources that are related to final goods production, whereas $\tilde{N}$ can be interpreted as human resources that are related to R&D activity. $B(\alpha, \gamma, \delta, \rho, \zeta)$ can be regarded as the threshold of level of R&D. The properties are as follows: it is obvious that $\frac{\partial B}{\partial \gamma} > 0$, $\frac{\partial B}{\partial \rho} > 0$ and $\frac{\partial B}{\partial \delta} < 0$. $\frac{\partial B}{\partial \alpha}$ and $\frac{\partial B}{\partial \zeta}$ are calculated as follows:

$$\frac{\partial B}{\partial \alpha} = -\frac{\gamma\rho\zeta(\alpha(1-\alpha) + (\zeta-\alpha))}{\delta\alpha^2(\zeta-\alpha)^2} < 0,$$

and

$$\frac{\partial B}{\partial \zeta} = -\frac{\gamma\rho\alpha(1-\alpha)}{\alpha\delta(\zeta-\alpha)^2} < 0.$$

DE is determined by the equation $\bar{s} = \arg_{s^*}\{\tilde{K}(s^*)^{RD} = \tilde{K}(s^*)^{KR}\}$. This equation provides one solution:

$$\bar{s} = \frac{(\alpha\delta(\zeta-\alpha)/\zeta)L + (1-\sigma)\delta\psi H - (1-\alpha)\gamma\rho}{[(1-\sigma)\delta\psi - (\alpha\delta(\zeta-\alpha)\gamma/\zeta)]H}.$$

When $\bar{s} \in (0,1)$, $\bar{s}$ is the equilibrium human capital share. The condition $0 < \bar{s} < 1$ is calculated as $\tilde{N} < B(\alpha,\gamma,\delta,\rho,\zeta) < N$ for $\sigma > \bar{\sigma}$, and $N < B(\alpha,\gamma,\delta,\rho,\zeta) < \tilde{N}$ for $\sigma < \bar{\sigma}$. It should be noted that $N < B(\alpha,\gamma,\delta,\rho,\zeta) < \tilde{N}$ contains the conditions of RE ($N < B$), and NE ($\tilde{N} > B$); therefore, RE and NE also become equilibria. Hence, this case generates ME. These results are summarized in Table. 1.

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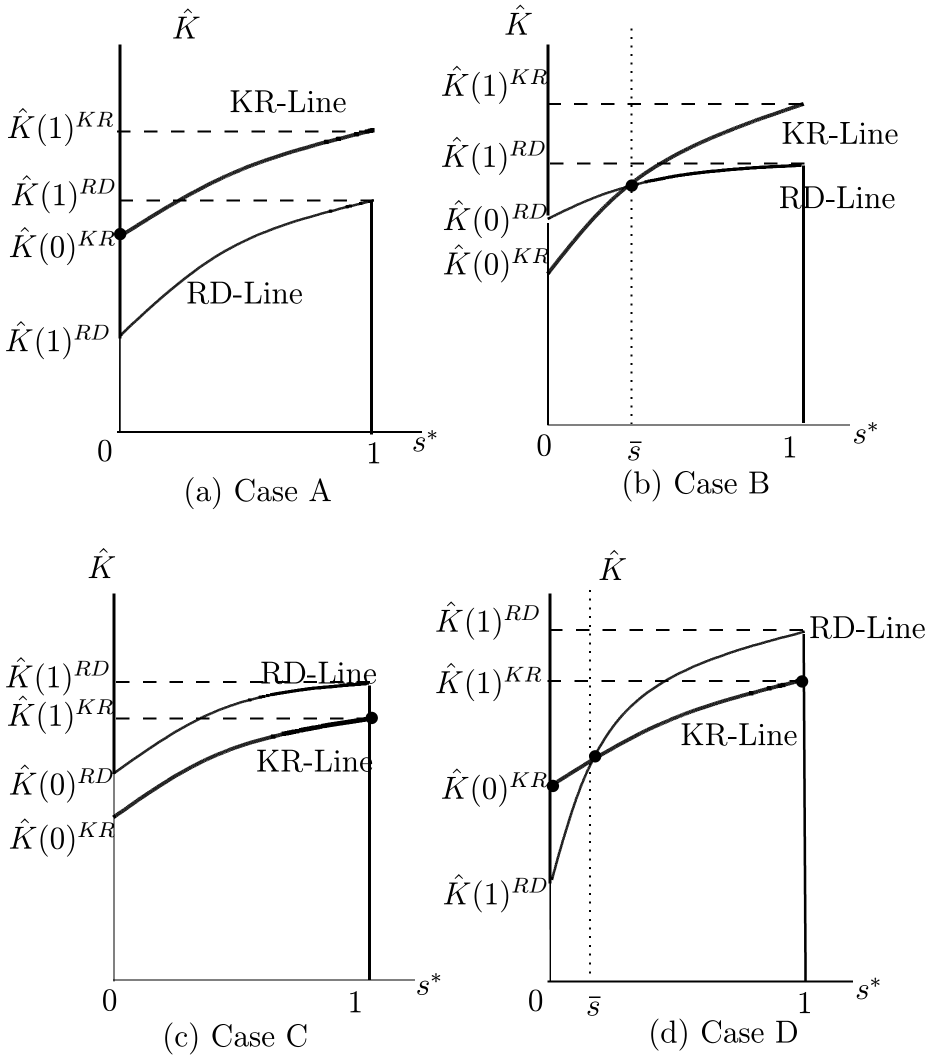
Table 1: Steady States and Equilibria

Equilibrium Type (Growth rate)	$s^* = 1$ ($g^* = 0$)	$s^* = \bar{s}$ ($g^* = \frac{\zeta-\alpha}{1-\alpha}\bar{s}^* H$)	$s^* = 0$ ($g^* = \frac{\zeta-\alpha}{1-\alpha} H$)	Condition about $B(\alpha, \delta, \rho, \zeta)$
Case A (RS)			◯	$B < N, \tilde{N}$
Case B (D)		◯		$\tilde{N} < B < N$
Case C (NG)	◯			$N, \tilde{N} < B$
Case D (ME)	◯	◯	◯	$N < B < \tilde{N}$

◯: Equilibrium



(a) Case A (b) Case B (c) Case C (d) Case D

Figure 1. Types of Steady States

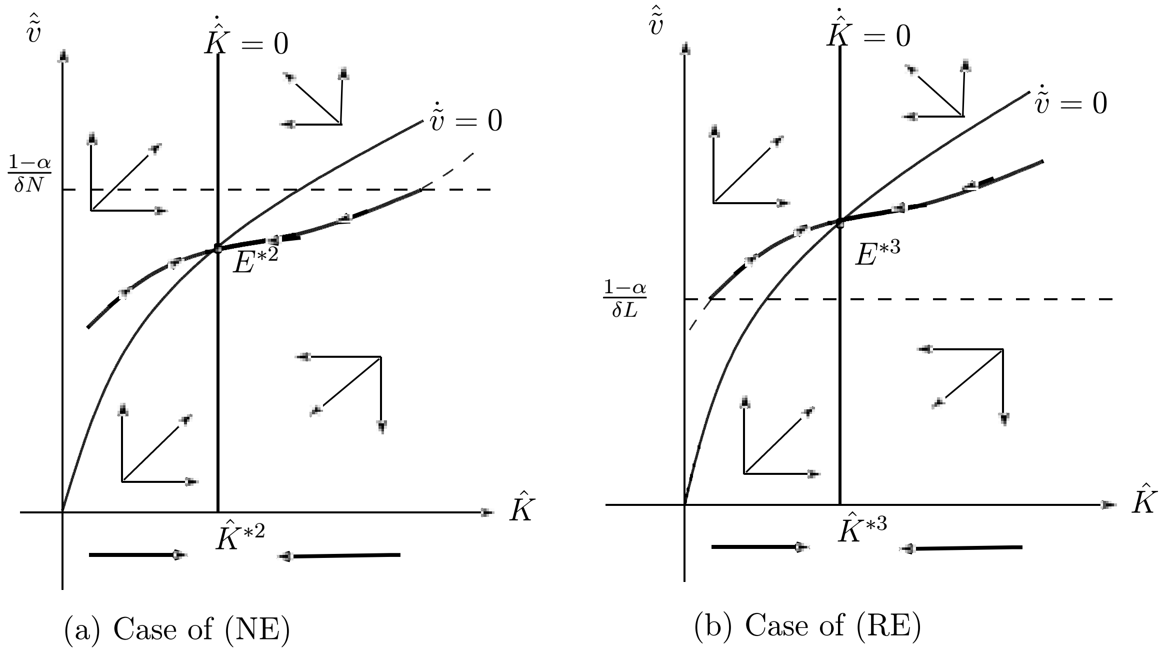


Figure 2. Phase Diagram of $(\hat{K}, \tilde{v})$