

# Polarization, Regime Switch and Economic Policies in the Process of Economic Development

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## Abstract

This study develops an endogenous growth model that depicts the process of economic development, such as a regime switch from capital-accumulation-based growth to R&D-based growth and the phenomenon falling into poverty traps. Furthermore, we also obtained several implications of economic policies, as follows. Interest rate subsidies promote economic welfare. Development aids through the provision of factor stocks such as capital and technology are ineffective for an economy to ride on a steady growth path. Promoting efficiency in goods production and/or R&D, on the other hand, is effective.

Keywords: Polarization of economies, R&D-based growth, poverty traps, regime switch of growth engine, effectivity of economic policies

JEL Classification: E00, O00, O41

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# 1 Introduction

This paper focuses on the issues of economic development and conducts a welfare analysis of economic policies for development. Large variations in economic growth rates have drawn the attention of many economists (e.g. Lucas 1988). Easterly (1994) and Quah (1996, 1997) have extracted the *polarization* of growth rates from this diversity. Furthermore, Princhett (1997) has reported that the divergence of economies can be observed in longer time spans (around two hundred years). Princhett's study noted that growth in developed countries has been constant in the long run, whereas the economies of the world's underdeveloped nations are caught in poverty traps regardless of the large amounts of domestic and international policy expenditure to foster economic development.

In this growth process, we can identify one property, i.e., the implications of *regime switch* on economic development. Some empirical studies show the change of growth engine on the processes of economic growth. Abramovitz and David (1973) demonstrated that in the early nineteenth century, America's economic growth was more heavily dependent on capital accumulation than on total factor productivity (TFP). Hayami and Ogasawara (1999) reported similar results from Japanese pre-war data. These works show that relatively developed economies grow as a result of capital accumulation at the early stage of development; later, these countries change their growth regime to one that is driven by research and development (R&D) activities. Since capital has a decreasing returns property and R&D activities perpetually increase the TFP, this regime change appears to be a critical event for long-run economic growth. The purpose of this study is to analyze the mechanics of the phenomena described above. Regime change and the realization of long-run growth have been receiving attention in this decade because of their connection with the endogenous growth theory. Some theoretical works, e.g., Zilibotti (1995), Matsuyama (1999), Funke and Strulik (2000), Galor and Moav (2004), Irmen (2005), and Kuwahara (2007), have developed models to describe the regime change from capital-based growth with decreasing returns to long-run positive growth. In particular, the present study has a similar aim, shared by Irmen (1995) and Kuwahara (2007), which concerns the analysis of regime switch from capital-based growth to R&D-based growth. In particular, the present study is consistent with those of Irmen (2005) and Kuwahara (2007) in terms of focusing on the regime

switch of economic growth from capital-based to R&D-based growth. While Irmen (2005) conducted the analysis using the model of competitive economy and Kuwahara (2007) develops a model with instantaneous monopoly power, this study constructs a model with permanently effective patent, therefore intertemporal effects of patent is analyzed.

By using an R&D-based growth model with capital as an input for R&D production, the present study aims to derive the cause of these phenomena of polarizing economies and regime change and to identify the theoretical condition for implementing effective policies for economic development. As is widely recognized among economists, the main source of economic growth is TFP and not capital accumulation. Economic growth endogenously derived by TFP has been incorporated into the first generation R&D-based growth models (e.g., Romer 1990, Grossman and Helpman 1991, and Aghion and Howitt 1992). These models assume that (exogenously endowed) human resources are used in R&D activities and that the long-run growth rate and R&D inputs are related, often concluding that the introduction of capital would not alter the essential results (e.g., Grossman and Helpman 1991 Ch.5 and Aghion and Howitt 1998 Ch.3). However, some studies have qualitatively related capital accumulation to R&D activities. For example, Abramovitz and David (1973) demonstrated that R&D activities positively depends on the level of capital accumulation, and Chandler (1990) demonstrated that the scale expansion of enterprises generates R&D activities and product diversification. Because an extent of economic development or a scale of firms can be interpreted as being reflected by capital accumulation, these studies imply the existence of a relationship between capital accumulation and R&D activities.

We utilize the model presented by Romer (1990). The main modification we make is that capital is used in R&D activities. This slight transformation of the assumption yields several remarkable results regarding the mechanism that engenders the polarization of economies and the implementation of effective economic policies for escaping poverty traps. First, the R&D-based growth model emphasizes the relationship between growth rates and R&D input endowment. In the first generation models of the R&D-based growth, the endowment of human resources, which are used in R&D activities and endowed exogenously, determine the economic growth rate. On the other hand, our assumption of capital R&D input relates the amount of the capital to economic growth rate. Furthermore, because the capital stock is

determined endogenously, our model provides the mechanism by which the long-run growth pattern is determined by the endogenously- determined endowment of R&D input factors. Therefore, our study delves further into the analysis of the causes of poverty traps. Moreover, our model relates the deep parameters of the model to long term growth rate through determining the equilibrium capital stock in the steady state. It indicates that lower costs of intermediate goods and higher R&D productivity are necessary to achieve positive long-run growth.

Second, we also derive some implications pertaining to the role of domestic and international policy in economic development. We identify the optimal growth path and examine economic welfare. Distortion of the intermediate goods sector causes the equilibrium capital stock to be excessively small. The GDP growth rate of the present model is positively related with capital. Therefore, a smaller capital stock engenders a lower GDP growth rate. The capital stock is stimulated by interest rate subsidies. Consequently, the steady-state growth rate is raised and optimal growth is realized. This optimal policy can enhance economic welfare, but it is unable to set the economy on a long-run growth path if the optimal path of an economy is a steady state with no growth. This result can easily be corresponded to the policy implication for official development assistance (ODA). Many studies, e.g., Easterly and Rebelo (1993) and Fischer (1993), conclude that economic growth in developing countries depends on their own economics policies. Boone (1996) insists that foreign aid has not raised the growth rates in poor countries, whereas Burnside and Dollar (2000) report that the effectiveness of an aid policy is conditional. Our results show that sound institutions are essential for economic growth, consistent with studies such as Hall and Jones (1999); Acemoglu, Johnson and Robinson (2001, 2004); and Dollar and Kraay (2003). Thus, empirical results can be theoretically obtained in the present study.

The paper is organized as follows: the model is established and the conditions of a decentralized economy are derived in Section 2. The existence of the two types of steady states and their determinants are presented in Section 3. The dynamic property of the model is analyzed in Section 4, and a welfare analysis is presented in Section 5. The paper is concluded in Section 6.

## 2 The Model

This study adopts a Romer-type (1990) production structure. Three sectors are considered in the present analysis: the final goods, intermediate goods, and R&D sectors. Three factors are used: labor, capital, and knowledge. Final goods are consumed as consumption goods or are invested as physical capital. They are produced with labor ( $L$ ), capital employed in the final goods sector ( $K_Y$ ), and a cluster of intermediate goods.<sup>1</sup> In this study, labor is employed only in the final goods sector, and capital can be used for final goods production ( $K_Y$ ) and investment to create new varieties of intermediate goods, namely R&D activities ( $K_A$ ). The market-clearing condition for capital imposes  $K = K_Y + K_A$ , where  $K$  is the total amount of capital in the economy. The production function of final goods is specified as

$$Y = L^{1-\alpha} \int_0^A x(i)^\alpha di, \quad 0 < \alpha < 1, \quad (1)$$

where  $Y$ ,  $L$ ,  $A$ , and  $x(i)$  indicate the final goods product, labor, the number of varieties, and  $i$ 's intermediate goods inputs, respectively. Intermediate goods are produced using physical capital and are used in the final goods production process. One unit of intermediate goods is assumed to be produced by  $\eta$  units of capital. Therefore capital devoted to final goods production  $K_Y$  is quantified as

$$K_Y \equiv \int_0^A \eta x(i) di. \quad (2)$$

An assumption of symmetric equilibrium regarding intermediate goods, that is  $x = x(i)$ , converts Eq. (2) into  $K_Y = \eta Ax$  or equivalently  $x = (1/\eta)(K_Y/A)$ . Substituting  $x(i) = x = (1/\eta)(K_Y/A)$  into Eq. (1) allows the following derivation:

$$Y = \eta^{-\alpha} L^{1-\alpha} A^{1-\alpha} K_Y^\alpha. \quad (3)$$

Because of the assumption that final goods  $Y$  are consumed or invested and Eq. (3), the following resource constraint of final goods holds:

$$\dot{K} = \eta^{-\alpha} L^{1-\alpha} A^{1-\alpha} K_Y^\alpha - C (= Y - C), \quad (4)$$

where  $\dot{K}$  and  $C$  denote the increment of aggregate capital  $K$  and aggregate consumption, respectively.

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<sup>1</sup>The scale of the cluster, that is, the variety of intermediate goods ( $A$ ), can be regarded as technological stock in this economy.

The final goods sector is competitive, Eq. (1) yields the first order conditions (FOCs) of final goods production that are given as  $\frac{\partial Y}{\partial L} = w$ , and  $\frac{\partial Y}{\partial x(i)} = p(i)$ , where  $w$  and  $p(i)$  denote the real wage and the price of  $i$ th sector intermediate goods, respectively.

The designs of intermediate goods are protected by patents. Therefore, intermediate goods are supplied monopolistically. In addition, a firm with a patent for  $i$ th intermediate goods production can be designated as a  $i$ th intermediate goods firm. As stated earlier, it is assumed that one unit of intermediate goods is produced using  $\eta$  units of capital. The profit of the  $i$ th intermediate goods sector is given as  $\pi(i) \equiv p(i)x(i) - r\eta x(i)$ , where  $r$  is the rental price of capital and  $\pi(i)$  is the profit of the  $i$ th intermediate goods firm. The intermediate goods firm maximizes this profit subject to  $\frac{\partial Y}{\partial x(i)} = p(i)$ . This optimization yields the following:

$$x(i) = x = \left(\frac{\alpha^2}{r\eta}\right)^{\frac{1}{1-\alpha}} L, \quad p(i) = p = \left(\frac{\eta}{\alpha}\right) r.$$

From Eqs. (1) and (2) and the FOCs, the market prices are obtained as

$$w = (1 - \alpha)\frac{Y}{L}, \quad r = \alpha^2\frac{Y}{K_Y}, \quad \text{and} \quad \pi = \pi(i) = \alpha(1 - \alpha)\frac{Y}{A}. \quad (5)$$

Innovation is assumed to be the discovery of a new design of intermediate goods that are added to the existing set of intermediate goods; therefore, the increment of new variety is the time differentiation of knowledge  $\dot{A}$ . In the process of innovation, since it is assumed that the input is capital, the firms undertake R&D by paying the rental cost  $r$ . R&D firms create the designs of new intermediate goods, and the patents of these designs bear the stream of monopoly profits  $\pi$ . The present value of this stream represents the value of R&D:

$$v_t \equiv \int_t^\infty \pi(\tau) e^{-\int_t^\tau r(s) ds} d\tau.$$

Thus, aggregate revenue and cost of R&D are given as  $v\dot{A}$  and  $rK_A$ , respectively.

Free entry of R&D is assumed; thus, if the profits from R&D are larger than the costs, then an infinite amount of capital would be allocated to R&D activities; this cannot hold in equilibrium. On the other hand, if the profit of R&D is less than its cost, then investment in R&D is unprofitable; thus, no resource is allocated to R&D, and an equilibrium without R&D ( $K_A =$

0) occurs. Thus, when the economy is in equilibrium with positive R&D activities, then the revenue of R&D must be equated to the cost of R&D. Thus, the relationships between market equilibrium and capital allocation are summarized as

$$\left. \begin{array}{l} \text{Solow Regime: } K_A = 0 \\ \text{Romer Regime: } K_A > 0 \\ \text{Not in equilibrium} \end{array} \right\} \iff rK_A \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} v\dot{A}, \quad (6)$$

Whether or not the economy conducts R&D depends on condition (6). When  $K_A > 0$ , R&D activities take place; this causes the economy to grow through endogenous technological change. Following Matsuyama (1999), we term this regime as the Romer regime. Condition (6) states that the equality holds in Romer regime. When  $K_A = 0$ , no R&D activity is operated, and therefore, the economy grows by only capital accumulation. Following Matsuyama (1999), we term this regime as the Solow regime. In the Romer regime, the system obey the equation in (6).

Following each regime, differentiating  $v$  with respect to time provides the following asset equations:

$$\left. \begin{array}{l} \text{Solow Regime: } K_A = 0 \\ \text{Romer Regime: } K_A > 0 \end{array} \right\} \iff rv \left\{ \begin{array}{l} > \\ = \end{array} \right\} \pi + \dot{v} \quad (7)$$

If R&D is undertaken, technological knowledge is assumed to increase according to

$$\dot{A} = \frac{\delta K_A}{L} = \delta(k - k_Y), \quad (8)$$

where  $k$  and  $k_Y$  respectively denote the per capita value of  $K$  and  $K_Y$  (more generally  $z$  denotes the per capita value of a aggregate variable  $Z$ ). In other words, the increment of knowledge depends on the capital investment devoted to R&D activities positively and population scale negatively. Furthermore, the both factors linearly affect the increment.

We examine the consumption decision to close the model. It is assumed that a representative household maximizes the additively separable utility function subject to a budget constraint:

$$U_t = \int_t^\infty \frac{c(\tau)^{1-\sigma} - 1}{1-\sigma} e^{-\rho(\tau-t)} d\tau, \quad \sigma > 0, \quad (9)$$

subject to

$$\dot{k} = rk + w - c - nk, \quad (10)$$

where  $\rho$ ,  $\sigma$ , and  $n$  denote the subjective discount rate, constant relative risk aversion (CRRA), and population growth rate, respectively. We assume that the population growth rate is exogenously constant. An optimal condition regarding consumption is the Keynes-Ramsey rule,

$$\sigma \frac{\dot{c}}{c} = r - \rho - n, \quad (11)$$

and the transversality condition

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) k(t) = 0, \quad (12)$$

where  $\lambda (\equiv c^{-\sigma})$  is the shadow price of capital stock  $K$ .

### 3 Steady States

Next, we analyze the economy in a steady state; here, all variables,  $Y$ ,  $C$ ,  $K$ ,  $K_Y$  and  $A$ , grow at constant rates (possibly zero). Our model contains two types of steady states. One is a steady state with R&D, therefore, positive growth, and the other is one with no R&D, therefore no growth. We term these steady states as the Romer Steady States (RSS) and the Solow Steady States (SSS), respectively.

Differentiating  $Y$  in Eq. (1) with respect to time yields  $g_Y = (1 - \alpha)(g_A + n) + \alpha g_K$ , where  $g_Z$  is the growth rate of variable  $Z$  i.e.,  $g_Z \equiv \dot{Z}/Z$ . Eq. (4) implies that  $g_Y = g_K = g_C$  in a steady state. These two conditions produce the following relation in a steady state:

$$g_Y^{ss} = g_K^{ss} = g_C^{ss} = g_A^{ss} + n, \quad (13)$$

where  $ss$  represents the value of steady states.

It is sometimes convenient to consider the variables of a constant in a steady state. Therefore, we define the knowledge-adjusted per capita value of a variable  $Z$  as  $\tilde{z} (\equiv Z/(AL))$ . Applying this notation on  $Y$ ,  $r$ ,  $\pi$  and  $g_A$ , we obtain the following:

$$\tilde{Y} = \eta^{-\alpha} \tilde{K}_Y^\alpha, \quad r = \alpha^2 \eta^{-\alpha} \tilde{K}_Y^{\alpha-1}, \quad \pi = (1 - \alpha) \alpha \eta^{-\alpha} \tilde{K}_Y^\alpha, \quad \text{and} \quad g_A = \delta(\tilde{K} - \tilde{K}_Y). \quad (14)$$

Substituting  $g_c^{ss} = g_A^{ss}$  into the Keynes-Ramsey rule of Eq. (11), we obtain steady state equations:

$$\sigma g_A^{ss} = r^{ss} - \rho - n. \quad (15)$$



$\pi(i)$  derived in Eq. (5) and the definition of  $v$  imply  $g_v^{ss} = g_\pi^{ss} = g_Y^{ss} - g_A^{ss}$ . Combining this and Eq. (13) yields  $g_\pi^{ss} = 0$ . Substituting  $\pi$  derived from Eq. (5),  $r$  derived from (15),  $g_A$  derived in Eq. (14), and  $g_v^{ss} = 0$  into (7), we obtain the following condition for R&D:

$$\left. \begin{array}{l} \text{Romer Steady States (RSS):} \\ \text{Solow Steady States (SSS):} \end{array} \right\} \iff \rho + n + \sigma\delta(\tilde{k}^{ss} - \tilde{k}_Y^{ss}) = \alpha^2\eta^{-\alpha}\tilde{k}_Y^{ss\alpha-1} \left\{ \begin{array}{l} = \\ > \end{array} \right\} \delta\frac{1-\alpha}{\alpha}\tilde{k}_Y^{ss} + n, \quad (16)$$

### 3.1 The property of the Romer Steady State (RSS)

In RSS, the equality in Eq. (16) holds and it determines the equilibrium capital allocation in the steady state between final goods production and R&D activities. From the second and third terms in Eq. (16), we obtain the equilibrium capital allocation to final goods production in RSS  $\tilde{K}_Y^*$  as

$$\tilde{k}_Y^* \equiv \arg \left\{ \tilde{k}_Y^{ss} \left| \alpha^2\eta^{-\alpha}\tilde{k}_Y^{ss\alpha-1} = \delta\frac{1-\alpha}{\alpha}\tilde{k}_Y^{ss} - n, \right. \right\} \quad (17)$$

Hereafter, we denote the value in RSS by indexing  $*$ .

From the first and third terms of Eq. (16), we can obtain the relationship between steady state knowledge-adjusted capital stock  $\tilde{k}^*$  and the steady state knowledge-adjusted capital stock devoted to final goods  $\tilde{k}_Y^*$ :

$$\tilde{k}^* = \frac{1}{\sigma} \left\{ \left( \sigma + \frac{1}{\alpha} - 1 \right) \tilde{k}_Y^* - \frac{\rho}{\delta} \right\}, \quad \text{or} \quad \tilde{k}_Y^* = \frac{\rho + \sigma\delta\tilde{k}^*}{\left( \sigma + \frac{1}{\alpha} - 1 \right) \delta}. \quad (18)$$

Substituting Eq. (18) into Eq. (14), we obtain the growth rate of the economy in RSS as<sup>2</sup>

$$g^* = \frac{1}{\sigma} \left( \delta\frac{1-\alpha}{\alpha}\tilde{k}_Y^* - \rho \right) = \frac{\left( \frac{1}{\alpha} - 1 \right) \delta\tilde{k}^* - \rho}{\sigma + \frac{1}{\alpha} - 1} \left( = \frac{\delta\tilde{k}^* - \frac{\alpha}{1-\alpha}\rho}{\frac{\alpha}{1-\alpha}\sigma + 1} \right). \quad (19)$$

<sup>2</sup>It is noteworthy that this growth rate closely resembles that of Eq. (13) in Romer (1990). Both equations commonly share the following properties: a higher R&D efficiency  $\delta$  and increased of R&D input (knowledge-adjusted capital stock  $\tilde{k}$  in the present study, and a higher human capital amount in Romer 1990); further, smaller values of the subjective discount rate  $\rho$  and CRRA parameter  $\sigma$  raises the economic growth rate. One difference between the two is as follows. We have set R&D input as capital, which is endogenously accumulated. Therefore, we can endogenously derive the R&D input factor in this study.

Eq. (19) implies that small  $\sigma$ ,  $\rho$  and  $\eta$  result in a faster growth rate in RSS.

From (19), the condition of positive growth is given as

$$\tilde{k}^* > \frac{\rho\alpha}{\delta(1-\alpha)} (\equiv \tilde{k}). \quad (20)$$

This implies sufficient capital in steady state is necessary for positive growth. Thus, the amount of capital is an important key determination of growth rate and appearance of poverty traps.

Substituting (18) into (17), we obtain the following equation which provides the knowledge-adjusted per capita capital in steady state:

$$n + \frac{\frac{1}{\alpha} - 1}{\sigma + \frac{1}{\alpha} - 1} \rho + \frac{\sigma(\frac{1}{\alpha} - 1)}{\sigma + \frac{1}{\alpha} - 1} \delta \tilde{k} = \alpha^2 \eta^{-\alpha} \left( \frac{\sigma + \frac{1}{\alpha} - 1}{\frac{\rho}{\delta} + \sigma \tilde{k}} \right)^{1-\alpha}. \quad (21)$$

From both sides of (21), we define the following equation.

$$\begin{aligned} L(\tilde{k}; \alpha, \delta, \rho, n, \sigma) &\equiv n + \frac{\frac{1}{\alpha} - 1}{\sigma + \frac{1}{\alpha} - 1} \rho + \frac{\sigma(\frac{1}{\alpha} - 1)}{\sigma + \frac{1}{\alpha} - 1} \delta \tilde{k} (= L(\tilde{k})) \\ R(\tilde{k}; \alpha, \delta, \eta, \rho, \sigma) &\equiv \alpha^2 \eta^{-\alpha} \left( \frac{\sigma + \frac{1}{\alpha} - 1}{\frac{\rho}{\delta} + \sigma \tilde{k}} \right)^{1-\alpha} (= R(\tilde{k})). \end{aligned}$$

$L(\tilde{k}; \alpha, \delta, \rho, n, \sigma)$  is an increasing line and  $R(\tilde{k}; \alpha, \eta, \delta, \rho, \sigma)$  is a decreasing curve with  $\lim_{\tilde{k} \rightarrow \infty} R(\tilde{k}) = 0$  and  $\lim_{\tilde{k} \rightarrow 0} R(\tilde{k}) = \infty$ . These two equations are drawn in Fig.1.

The properties of steady growth path in decentralized economy requires the following equation.

$$\tilde{k}^* = \arg\{\tilde{k} \mid L(\tilde{k}; \alpha, \delta, \rho, n, \sigma) = R(\tilde{k}; \alpha, \eta, \delta, \rho, \sigma)\}, \quad (22)$$

where  $\tilde{k}^*$  is the knowledge-adjusted per capita capital stock in steady growth path. Totally differentiating the both sides of (22) with respect to  $\tilde{k}$  and  $\chi (\in \{\alpha, \delta, \eta, \rho, n, \sigma\})$  yields the following derivative:

$$\frac{d\tilde{k}}{d\chi} = - \frac{\frac{\partial L}{\partial \chi} - \frac{\partial R}{\partial \chi}}{\frac{\partial L}{\partial \tilde{k}} - \frac{\partial R}{\partial \tilde{k}}}. \quad (23)$$

Differentiating  $L(\tilde{k}; \alpha, \delta, \rho, n, \sigma)$  and  $R(\tilde{k}; \alpha, \eta, \delta, \rho, \sigma)$  with respect to  $\tilde{k}$ ,  $\delta$ ,  $\eta$ ,  $n$  and  $\rho$ , we obtain the following results;  $L_{\tilde{k}} > 0$ ,  $L_{\delta} > 0$ ,  $L_{\eta} = 0$ ,  $L_{\rho} > 0$ ,

$L_n > 0$ ,  $R_{\tilde{k}} < 0$ ,  $R_\delta > 0$ ,  $R_\eta < 0$ ,  $R_\rho < 0$ , and  $R_n = 0$ . These derivatives yield the followings:

$$\frac{d\tilde{k}^*}{d\eta} < 0, \quad \frac{d\tilde{k}^*}{d\rho} < 0, \quad \text{and} \quad \frac{d\tilde{k}^*}{dn} < 0. \quad (24)$$

The sign of  $\frac{d\tilde{k}^*}{d\delta}$  is ambiguous. These results insist that lower cost of intermediate goods, lower subjective discount rate and lower population growth rate is necessary for larger knowledge-adjusted per capita capital stock.  $d\tilde{k}^*d\rho < 0$  and (19) show that population growth is negatively related on economic growth. It is the opposite properties which is presented by semi-endogenous growth models with labor R&D inputs.

From (20), the condition of positive growth is given as

$$\tilde{k}^* > \underline{\tilde{k}}. \quad (25)$$

Because  $L(\tilde{k})$  and  $R(\tilde{k})$  are monotonous increasing and decreasing function, respectively, the condition (25) is equivalent to

$$L(\underline{\tilde{k}}) < R(\underline{\tilde{k}}). \quad (26)$$

Substituting  $\underline{\tilde{k}}$  into  $L(\tilde{k})$  and  $R(\tilde{k})$ , we now study the condition required in RSS, which is characterized as  $g^* > 0$ .

$$\text{Parameter Condition of an RSS: } n + \rho < \Omega(\rho; \alpha, \eta, \delta), \quad (27)$$

where  $\Omega \equiv \alpha^{1+\alpha}\eta^{-\alpha} \left( \frac{\delta(1-\alpha)}{\rho} \right)^{1-\alpha}$ . Small  $n$ , small  $\rho$  and large  $\delta$  realize this case.

### 3.2 The property of the Solow Steady State (SSS)

Secondly, we derive the steady state capital stock in the SSS. In the SSS, since (16) holds with inequality, all capital is devoted to final goods production; therefore,  $\tilde{k}_A^{**} = 0$ , namely,  $\tilde{k}^{**} = \tilde{k}_Y^{**}$  where  $**$  denotes the steady state value in SSS. Eq. (4) and  $g_A = 0$  imply that the growth rate in this case is given as  $g_K = g_Y = g_C \equiv g^{**} = n$ . Substituting  $g_C = n$ , therefore  $g_c = 0$  into Eq. (11) yields  $r = \rho + n$ . Eq. (5) results in  $r = \alpha^2\eta^{-\alpha}\tilde{k}^{\alpha-1} + n$ , and Eq. (11) and  $g_c = 0$  yield  $r - \rho - n = 0$ . From these two equations, equilibrium capital stock in SSS is given as follows.

$$\text{No Growth Equilibrium: } \tilde{k}^{**} = \tilde{k}_Y^{**} = \left[ \frac{\alpha^2\eta^{-\alpha}}{\rho + n} \right]^{\frac{1}{1-\alpha}}. \quad (28)$$

Eq. (28) implies that when  $\rho$  and  $\eta$  are large, they bear a small amount of equilibrium capital stock. From the second and third terms of (16) and Eq. (28), we have the following inequality which must hold in SSS:

$$\alpha^2 \eta^{-\alpha} \left[ \frac{\alpha^2 \eta^{-\alpha}}{\rho + n} \right]^{-1} > \delta \frac{1 - \alpha}{\alpha} \left[ \frac{\alpha^2 \eta^{-\alpha}}{\rho + n} \right]^{\frac{1}{1-\alpha}} + n.$$

Solving this inequality, we can obtain the condition for the SSS as

$$\text{Parameter Condition of an SSS: } \rho + n > \Omega(\rho; \alpha, \eta, \delta). \quad (29)$$

Large  $n$  and  $\rho$  and small  $\delta$  cause this case.

### 3.3 The Determination of Steady State

The conditions of Eq. (27) and those of Eq. (29) are mutually exclusive. Therefore, the determination of the steady state is summarized in the following proposition.

**Proposition 1:** *An economy has a unique long-run steady state of growth or poverty traps that is determined by the following condition:*

$$\rho + n \begin{cases} < \\ > \end{cases} \Omega(\rho; \alpha, \eta, \delta) \Leftrightarrow \begin{cases} \text{RSS} \\ \text{SSS} \end{cases}$$

*This condition shows that the parameter set  $\{\alpha, \eta, \rho, \delta\}$  uniquely determines either steady state. Thus, deep parameters determine the growth rate in the long-run.*

The relationship is drawn in Figure 2.  $n + \rho$  is a linear increasing function and  $\Omega$  is a monotonously decreasing function and  $\lim_{\rho \rightarrow 0} \Omega(\rho) = \infty$ , both lines have only one solution written as  $\underline{\rho}$ . This  $\underline{\rho}$  determines the upper bound of the subjective discount rate  $\rho$  for steady growth. If  $\rho$  is smaller than  $\underline{\rho}$ , the economy has the steady state with a steady growth, and if  $\rho$  is larger, the economy has a poverty traps. The increase of population growth rate  $n$  shifts  $n + \rho$  upper and  $\Omega$  is invariant for the change of  $n$ , therefore, the increase of  $n$  declines the value of  $\underline{\rho}$ . From the definition of  $\Omega$ ,  $\Omega_\eta < 0$ , and  $\Omega_\delta > 0$ , an increase of  $\eta$  and a decrease of  $\delta$  declines the value of  $\underline{\rho}$ .

## 4 Transition Dynamics and Steady States

### 4.1 Local Transition dynamics

A decentralized economic system comprises Eqs.(4), (6), (7), (11), and (14). The system is reconstructed into the system constituted by  $\tilde{k}$ ,  $\tilde{C}$  and  $\tilde{k}_Y$ . Substituting  $\tilde{Y}$  and  $r$  given in Eq. (14) into Eq. (4), we obtain:

$$\dot{\tilde{k}}(t) = \eta^{-\alpha} \tilde{k}_Y(t)^\alpha - \tilde{c}(t) - \delta(\tilde{k}(t) - \tilde{k}_Y(t))\tilde{k}(t). \quad (30)$$

Substituting  $g_{\tilde{c}} + g_A = g_c$  and  $r = \alpha^2 \eta^{-\alpha} \tilde{k}_Y^{\alpha-1}$  in Eq. (11), we obtain the dynamics of  $\tilde{c}$  as follows:

$$\dot{\tilde{c}}(t) = \frac{1}{\sigma} \{ \alpha^2 \eta^{-\alpha} \tilde{k}_Y(t)^{\alpha-1} - \rho - n - \sigma \delta(\tilde{k}(t) - \tilde{k}_Y(t)) \} \tilde{c}(t). \quad (31)$$

Regarding the dynamics of  $\tilde{k}_Y$ , each regime follows different dynamics described below.

#### 4.1.1 Dynamics of the economy in the Romer Regime

Then, uniting  $g_v = r - \frac{\delta\pi}{rL}$  (from the arbitrage condition require (7) and free entry into R&D (6) and  $g_r = g_v - n$  (from the free entry into R&D (6)), and eliminating  $g_v$  from them, we obtain  $g_r + n = r - \frac{\delta\pi}{rL}$ .  $r = \alpha^2 \eta^{-\alpha} \tilde{k}_Y^{\alpha-1}$  derives  $g_r = (\alpha - 1)g_{\tilde{k}_Y}$  and  $\delta\pi/(rL) = \delta(1 - \alpha)\tilde{k}_Y/\alpha$ . Substitution of these equations into  $g_r + n = r - \frac{\delta\pi}{rL}$  yields the dynamics of  $\tilde{k}_Y$  as

$$\dot{\tilde{k}}_Y(t) = \frac{\delta}{\alpha} \tilde{k}_Y(t)^2 + \frac{n}{1 - \alpha} \tilde{k}_Y(t) - \frac{\alpha^2 \eta^{-\alpha}}{1 - \alpha} \tilde{k}_Y(t)^\alpha. \quad (32)$$

Because the dynamics of  $\tilde{k}_Y$  guided by Eq. (32) is the function that contains only  $\tilde{k}_Y$  as a variable, the dynamic properties of  $\tilde{k}_Y$  are immediately obtained from Eq. (32). The dynamics of  $\tilde{k}_Y$  are easily found to be unstable; the phase diagram of  $\tilde{k}_Y$  is given in Figure 3. Therefore, for the realization of RSS, it is necessary that  $\tilde{k}_Y(t) = \tilde{k}_Y^*$  must be satisfied in, at least, the neighborhood of the steady state. If  $\tilde{k}_Y(t) < \tilde{k}_Y^*$ ,  $\tilde{k}_Y(t)$  decreases following the dynamics of Eq. (32).

**Lemma 1** *On the transition path converging to RSS,  $\tilde{k}_Y$  must be constant at  $\tilde{k}_Y^*$  in the neighborhood of RSS.*

Thus, the dynamical system in the Romer regime must ride on the plain  $\tilde{k}_Y(t) = \tilde{k}_Y^*$ , that we term the "Romer regime manifold." Consequently, the system is reduced into the two-dimensional system comprising  $\tilde{k}$  and  $\tilde{C}$ . Because the system displays properties similar to the dynamic property of the Ramsey model, it is easily verified that this system has saddle stability.

#### 4.1.2 Dynamics of the economy in the Solow Regime

In this regime,  $\tilde{k}_A = 0$  leads to  $\tilde{k}(t) = \tilde{k}_Y(t)$ . Thus, the Solow regime also exists on the two-dimensional plane, which we call the Solow regime manifold. Under this condition, the system comprising (30) and (31) is made into

$$\dot{\tilde{k}}(t) = \eta^{-\alpha} \tilde{k}(t)^\alpha - \tilde{c}(t) - n\tilde{k}(t), \quad (33)$$

$$\dot{\tilde{c}}(t) = \frac{1}{\sigma} \{ \alpha^2 \eta^{-\alpha} \tilde{k}(t)^{\alpha-1} - \rho - n \} \tilde{c}(t). \quad (34)$$

Thus, the dynamical system in this case is similar to that of the normal Solow model; one difference is the interest rate. Because an *à la* Romer type of R&D-based growth model contains distortional intermediate goods pricing and our model assumes that intermediate goods are made by capital, interest rate (equivalently capital rental price) is  $\alpha$  times smaller than the normal Solow model.

## 4.2 Global Transitional Dynamics and Steady States

Uniting the local transition dynamics discussed in the previous section, we derive the global dynamics in this section. Depending on the two types of steady state, RSS and SSS, we will obtain the two growth patterns.

From Proposition 1, if an economy has RSS (namely,  $\rho + n < \Omega$ ), then RSS is the economy's unique steady state. In this case, Eq. (32), and consequently  $\tilde{k}_Y = \tilde{k}_Y^*$ , must hold in the steady state and the neighborhood. From Section 4.1.1, we observe that the system is saddle stable on  $\tilde{k} - \tilde{c}$  plane. The phase diagram of this regime is drawn on the dotted region in Figure 4. If initial knowledge-adjusted capital stock  $\tilde{k}(0)$  is larger than  $\tilde{k}_Y^*$ ,  $\tilde{k}_Y$  remains at  $\tilde{k}_Y^*$  and the system comprising  $\{\tilde{k}, \tilde{c}\}$  converges into RSS. If  $\tilde{k}(0)$  is smaller than  $\tilde{k}_Y^*$ , then resource constraint induces the R&D activity to ride on the Romer regime manifold, such that the economy rides on the Solow regime manifold and grows through capital accumulation. The Solow

regime is drawn on the shaded region in Figure 4. After sufficient knowledge-adjusted capital stock is accumulated, the economy begins R&D activities; thus, the economy experiences a regime switch from the Solow regime to the Romer regime.

On the other hand, if an economy has SSS (namely,  $\rho > \Omega$ ), Proposition 1 implies that SSS is the economy's unique steady state. In this case, even if a plentiful initial capital or knowledge stock exists in the economy, the economy fails to execute R&D. This mechanism is as follows. Because steady state knowledge-adjusted capital stock is smaller than the capital stock that effects a balance between goods production and R&D, Eq. (32)—that is derived from the arbitrage condition between capital and the R&D firm—cannot hold after a finite period in the future. This expectation leads rational agent to refrain from investing in R&D activity. Thus, this economy grows without R&D, and falls into SSS. This is the mechanism of the no-growth trap in this study. Some medium developed countries demonstrate failure to transit long-term positive growth in the polarization process; the mechanism presented here may be the cause of this phenomenon. The phase diagram of this economy is depicted in Figure 5.

**Proposition 2** *An economy has a unique steady state and a perfect foresight saddle-stable transition path that is convergent with the steady state. The long-run growth phase, showing either steady growth or poverty traps, is determined uniquely according to technological parameters ( $\alpha, \eta$  and  $\delta$ ) and preference parameter ( $\rho$ ). The economy with RSS (and low initial capital endowment) experiences a regime switch from capital-accumulation-based growth to R&D-based growth and realizes long-run growth. The economy with SSS lacks the profitability for R&D investment and persistently stays in Solow regime and is thus caught in poverty traps.*

## 5 Optimal Growth and Economic Policy

The previous section shows the results of the present model with regard to decentralized economic growth. Following the perfect foresight path determined by given parameters, an economy is convergent to steady state with long-run growth or no growth. However, the present model contains a distortion in the intermediate goods pricing and a bad equilibrium termed as SSS.

This section examines the possible roles of the government through economic policies on economic welfare and development.

## 5.1 Command Economy

To obtain the welfare properties of a decentralized solution, consider the social planner formulation of this growth model. A benevolent government is assumed to maximize the representative household's utility function Eq. (9). Therefore, a Hamiltonian of the government is written as

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda \underbrace{(\eta^{-\alpha} k_Y^\alpha A^{1-\alpha})}_y - c - nk + \mu \delta(k - k_Y),$$

where  $\lambda$  and  $\mu$  are the shadow prices of per capita capital stock and knowledge, respectively. The optimal conditions are obtained as follows:

$$\lambda = c^{-\sigma}, \quad (35)$$

$$\lambda \alpha \eta^{-\alpha} k_Y^{\alpha-1} A^{1-\alpha} = \mu \delta, \quad (36)$$

$$\rho \lambda - \dot{\lambda} = \frac{\partial \mathcal{H}}{\partial k} = -\mu \delta, \quad (37)$$

$$\rho \mu - \dot{\mu} = \frac{\partial \mathcal{H}}{\partial A} = \mu(1-\alpha) \eta^{-\alpha} k_Y^\alpha A^{-\alpha}, \quad (38)$$

Using Eqs. (35), (37), and (38), we derive the following equations:

$$\rho - g_\lambda = \alpha \eta^{-\alpha} k_Y^{\alpha-1} A^{1-\alpha}, \quad (39)$$

$$\rho - g_\mu = \frac{1-\alpha}{\alpha} \frac{\delta k_Y}{A}, \quad (40)$$

From Eqs. (35) and (39) and using the definition of  $\tilde{c}$  and  $\tilde{k}$ , the following Euler equation is produced as

$$g_{\tilde{c}} = \frac{1}{\sigma} (\alpha \eta^{-\alpha} \tilde{k}_Y^{\alpha-1} - \rho - n - \sigma \delta(\tilde{k} - \tilde{k}_Y)). \quad (41)$$

Because Eq. (36) yields  $g_\mu = g_\lambda + g_y - g_{k_Y}$  and Eq. (39) is converted into  $g_\lambda = \rho - \alpha \eta^{-\alpha} \tilde{k}_Y^{\alpha-1}$ ,  $g_\mu$  is derived as

$$g_\mu = \rho - \alpha \eta^{-\alpha} \tilde{k}_Y^{\alpha-1} + g_y - g_{k_Y}. \quad (42)$$



Substituting Eq. (42) into Eq. (40), we obtain the optimal capital allocation dynamics as follows:  $\alpha\eta^{-\alpha}\tilde{k}_Y^{\alpha-1} - g_Y + g_{k_Y} = \frac{1-\alpha}{\alpha}\delta\tilde{k}_Y$ . Using  $\tilde{k}_Y$ ,  $\tilde{k}$ , and  $g_Y = \alpha g_{k_Y} + (1-\alpha)g_A$ , and differentiating with Respect with  $\tilde{k}_Y$ , we get

$$g_{\tilde{k}_Y} = g_{k_Y} - g_A = \frac{1}{1-\alpha} \left[ \frac{\delta(1-\alpha)}{\alpha}\tilde{k}_Y - \alpha\eta^{-\alpha}\tilde{k}_Y^{\alpha-1} \right]. \quad (43)$$

The system consists of these three dynamic equations: (4), (41), and (43), which imply that  $g_{K_Y} = g_K$  and  $g_A = g_k = g_c = \delta(k - k_Y)/A$  must hold in a steady state. Rewriting these conditions with  $\tilde{k}$ ,  $\tilde{y}$ , and  $\tilde{c}$ , we obtain  $g_{\tilde{k}_Y} = g_{\tilde{k}}$  and  $g_A = g_k = g_c = \delta(\tilde{k} - \tilde{k}_Y)$ . Substituting these conditions into Eqs. (4), (41), and (43), we obtain the following equations depicting the steady state of the command economy:

$$\rho + \sigma\delta(\tilde{k}^{*op} - \tilde{k}_Y^{*op}) = \alpha\eta^{-\alpha}\tilde{k}_Y^{*op\alpha-1} - n, \quad (44)$$

$$\eta^{-\alpha}\tilde{k}_Y^{*op\alpha} - \tilde{c}^{*op} - \{n + \delta(\tilde{k}^{*op} - \tilde{k}_Y^{*op})\}\tilde{k}^{*op} = 0 \quad (45)$$

$$\alpha\eta^{-\alpha}\tilde{k}_Y^{*op\alpha-1} - n = \frac{1-\alpha}{\alpha}\delta\tilde{k}_Y^{*op}. \quad (46)$$

Eliminating  $\alpha\frac{\tilde{y}}{u\tilde{k}} - n$  from (44) and (46), we obtain the following equation:  $\rho + \sigma\delta(\tilde{k} - \tilde{k}_Y) = \frac{1-\alpha}{\alpha}\delta\tilde{k}_Y$ . Solving this equation with respect to  $u$ , we obtain the optimal capital allocation in steady state as

$$\tilde{k}_Y^{*op} = \frac{\rho + \sigma\delta\tilde{k}^{*op}}{(\sigma + \frac{1}{\alpha} - 1)\delta}. \quad (47)$$

From (14) and (47), the optimal growth rate of economy is given as

$$g^{*op} = \frac{1}{\sigma} \left( \delta\frac{1-\alpha}{\alpha}\tilde{k}_Y - \rho \right) = \frac{(\frac{1}{\alpha} - 1)\delta\tilde{k}^{*op} - \rho}{\sigma + \frac{1}{\alpha} - 1}. \quad (48)$$

$g^*$  insists that a higher subjective discount rate  $\rho$ , a higher R&D efficiency  $\delta$ , a knowledge-adjusted per capita capital stock  $\tilde{k}$ , and a smaller CRRA parameter  $\sigma$  make the growth rate of economy higher.

Substituting  $u^{*op}$  and  $g^{*op}$  into (44), we obtain the following equation which provides the knowledge-adjusted per capita capital in steady state:

$$n + \frac{\frac{1}{\alpha} - 1}{\sigma + \frac{1}{\alpha} - 1}\rho + \frac{\sigma(\frac{1}{\alpha} - 1)}{\sigma + \frac{1}{\alpha} - 1}\delta\tilde{k}^{*op} = \alpha\eta^{-\alpha} \left( \frac{\sigma + \frac{1}{\alpha} - 1}{\frac{\rho}{\delta} + \sigma\tilde{k}^{*op}} \right)^{1-\alpha}. \quad (49)$$

From both sides of (49), we define the following equation.

$$L(\tilde{k}; \alpha, \delta, \rho, n, \sigma) \equiv n + \frac{\frac{1}{\alpha} - 1}{\sigma + \frac{1}{\alpha} - 1} \rho + \frac{\sigma \left(\frac{1}{\alpha} - 1\right)}{\sigma + \frac{1}{\alpha} - 1} \delta \tilde{k}^{op}$$

$$R^{op}(\tilde{k}; \alpha, \delta, \eta, \rho, \sigma) \equiv \alpha \eta^{-\alpha} \left( \frac{\sigma + \frac{1}{\alpha} - 1}{\frac{\rho}{\delta} + \sigma \tilde{k}^{op}} \right)^{1-\alpha} \quad (= R^{op}(\tilde{k})).$$

$L(\tilde{k}; \alpha, \delta, \rho, n, \sigma)$  is the identical of the LHS of Eq.(21) and  $R^{op}(\tilde{k}; \alpha, \eta, \delta, \rho, \sigma)$  is  $\alpha$  times for RHS of Eq.(21)  $R(\tilde{k}; \alpha, \eta, \delta, \rho, \sigma)$ , then,  $R^{op}$  is also drawn in Fig.1. The properties of steady growth path in the command economy requires the following equation.

$$\tilde{k}^{*op} = \text{arg}\{\tilde{k} \mid L(\tilde{k}; \alpha, \delta, \rho, n, \sigma) = R^{op}(\tilde{k}; \alpha, \eta, \delta, \rho, \sigma)\}, \quad (50)$$

where  $\tilde{k}^{*op}$  is the steady-growth knowledge-adjusted per capita capital stock in command economy. Because the difference between decentralized economy and command economy is that  $R$  is  $\alpha$  times  $R^{op}$ , the properties of steady state are similar to each other. In steady state of optimal economy, lower  $\rho$ ,  $\eta$ , and  $n$  make a higher growth rate of per capita GDP growth rate. These are common to the steady state in the decentralized economy. However, the equilibrium capital stock of command economy is larger than that of decentralized economy, therefore, the growth rate of this economy is higher than that of decentralized economy.

This implies sufficient capital is necessary for steady growth. If this condition is lacked, no R&D investment is optimal, then  $K_A = g_A = 0$ , and the poverty-trap knowledge-adjusted per capita capital stock in command economy  $\tilde{k}^{**op}$  is given as

$$\tilde{k}^{**op} = \left[ \frac{\alpha \eta^{-\alpha}}{n + \rho} \right]^{\frac{1}{1-\alpha}}. \quad (51)$$

From (47) and (48), the condition of feasible positive growth  $g^*(\tilde{k}^{op}) > 0$  is given as

$$\tilde{k}^{op} > \underline{\tilde{k}}. \quad (52)$$

Because  $L(\tilde{k})$  and  $R^{op}(\tilde{k})$  are monotonous increasing and decreasing function, respectively, the condition (52) is equivalent to

$$L(\underline{\tilde{k}}) < R^{op}(\underline{\tilde{k}}). \quad (53)$$

Substituting  $\tilde{k}$  into  $L(\tilde{k})$  and  $R(\tilde{k})$ , we obtain the following inequality:

$$n + \rho < \Omega^{op}(\rho; \alpha, \eta, \delta), \quad (54)$$

where  $\Omega^{op} \equiv \alpha^\alpha \eta^{-\alpha} \left( \frac{\delta(1-\alpha)}{\rho} \right)^{1-\alpha}$  ( $= \frac{1}{\alpha} \Omega(\rho)$ ). Small  $n$ , small  $\rho$  and large  $\delta$  realize this case. Uniting (50) and (51), we obtain the equilibrium capital stock in command economy  $\tilde{k}^{op}$  as follows:

**Proposition 1’:** *An economy has a unique optimal long-run steady state of positive growth or no growth traps that is determined by the following condition:*

$$\tilde{k}^{op} = \begin{cases} \tilde{k}^{*op} & \text{if } n + \rho < \Omega^{op} \\ \tilde{k}^{**op} & \text{if } n + \rho > \Omega^{op} \end{cases}$$

$$\Rightarrow \text{Long-run optimal growth is } \begin{cases} \text{positive growth (RSS)} \\ \text{no growth (SSS)} \end{cases}, \quad (55)$$

where  $\Omega^{op} \equiv \alpha^\alpha \eta^{-\alpha} \left( \frac{\delta(1-\alpha)}{\rho} \right)^{1-\alpha}$  ( $= \frac{1}{\alpha} \Omega(\rho)$ ). A small  $\rho$  and  $n$  and large  $\delta$  realize this case.

## 5.2 Effects of Economic Policies

Here, taxes and subsidies are introduced into our model. It is proposed that a constant rate subsidy  $s > 0$  (a tax if  $s < 0$ ) is levied (provided) for interest (rental price of capital) and the profit of the intermediate sector, as below

$$r^s \equiv (1 + s_r)r = (1 + s_r)\alpha^2 \frac{Y}{K_Y},$$

$$\pi^s \equiv (1 + s_\pi)\pi = (1 + s_\pi)\alpha(1 - \alpha) \frac{Y}{A},$$

where  $s_r$  and  $s_\pi$  represent the interest and a profit subsidies rate, respectively.

The existence of distortion in the intermediate goods market leads the decentralized economy to accumulate less knowledge-adjusted stock of capital than the command economy. For this reason, an economic policy to promote capital accumulation by subsidizing the interest rate always improves economic welfare.

The government is assumed to finance these subsidies using lump-sum tax revenues. The total tax revenue is expressed as  $T^{LS}$ . We assume that

the government maintains a balanced financial policy: the budget constraint  $s_r K + s_\pi A = T^{LS}$  is always satisfied. Translating  $r$  and  $\pi$  in Eqs. (31) and (32) to  $r^s$  and  $\pi^s$ , we determine both lines after taxation (or subsidy) as follows.

$$\dot{\tilde{c}} = \frac{1}{\sigma} \left\{ (1 + s_r) \alpha^2 \eta^{-\alpha} \tilde{k}_Y^{\alpha-1} - \rho - n - \sigma \delta (\tilde{k} - \tilde{k}_Y) \right\} \tilde{c}, \quad (56)$$

$$\dot{\tilde{k}}_Y = \frac{\delta}{\alpha} (1 + s_\pi) \tilde{k}_Y^2 + \frac{n}{1 - \alpha} \tilde{k}_Y - (1 + s_r) \frac{\alpha^2 \eta^{-\alpha}}{1 - \alpha} \tilde{k}_Y^\alpha. \quad (57)$$

For optimal growth, Eqs. (56) and (57) must correspond with Eqs. (41) and (43) (in Appendix), respectively.

**Lemma 2:** *Optimal growth rate and capital allocation are realized by the following subsidy policies:*

$$s_r^* = \frac{1}{\alpha} - 1 > 0, \quad s_\pi^* = 0.$$

Because  $\alpha \in (0, 1)$ ,  $s_r^*$  is always constant and negative, implying that this effective policy is perennial and that it increases the welfare of the economy.

### 5.3 Economic Policy for "Take-off"

The previous section showed that an economic policy of interest-rate subsidies can increase the welfare of the economy by equalizing a decentralized economy to the Pareto-efficient economy derived in Section 5.1. On the one hand, can an optimal subsidy policy thrust a country from a poverty trap into steady growth? The answer, at least partially, is in the affirmative.

As is derived in Section 5.1, the Pareto-efficient steady state is given by  $\tilde{k}^{ssop}$ , and this value does not always satisfy the condition for positive growth given by Eq. (55). If  $\tilde{k}^{ssop} < \tilde{k}$ , the optimal steady state is that of no-growth. Therefore, the no-growth equilibrium (SSS) is an optimal path for the country in this case. If the government desires long-run positive growth in such a case, some parameters must be relevantly promoted; for example, it is necessary to increase the R&D efficiency  $\delta$ .

If the parameter set of a country yields  $\tilde{k}^{ssop} > \tilde{k}$ , the optimal steady state of the economy is long-run positive growth (RSS). However, the decentralized economy of the county will be caught in SSS if the country has, in addition, a parameter set that causes  $\tilde{k}^{**} < \tilde{k} (< \tilde{k}^{ssop})$  (which yields the condition  $\Omega <$

$\rho < \Omega^{op}$  from Proposition 1 and 1'); thus, a no-subsidy policy is executed. In this case, the country potentially possesses the capability for long-run positive growth, but monopoly power exercised in the decentralized economy draws the economy into a no-growth trap. In this situation, the optimal subsidizing policy for canceling out the monopoly pricing transforms the long-run steady state from SSS into RSS.

In the case of  $\tilde{k}^{ss\,op} < \tilde{k}$ , the economy has no optimal path with a positive long-run growth, and the economy cannot possibly ride on a steady growth path without harming the welfare of the country. Therefore, some external economic aids are necessary for realizing positive long-run growth in this country. Hence, we demonstrate the effects of ODA, which refers to financial or technological aid offered by developed countries to their underdeveloped counterparts (i.e., countries with SSS under the optimal policies) for the purpose of launching the countries on a steady growth path, namely, the Romer regime.

Generally speaking, the ODA measures of licensing technology and providing capital stock are regarded as increments of  $A$  and  $K$ , respectively. These produce effects on the endowment of knowledge-adjusted capital  $\tilde{k}$ . However, the equilibrium conditions of neither Proposition 1 nor 1', steady states of the aided countries remain wholly unaffected. Consequently, the economy will continue to be on a no-growth path; the economy consumes capital to converge to SSS. These effects can be considered to only jump to a point on the long-run no-growth path. Therefore, for effective ODA, it is necessary to aid an economy to change the conditions of Proposition 1'. Therefore, the ODA should improve an efficiency parameter such as the cost of intermediate goods production  $\eta$  or R&D efficiency  $\delta$ . To ensure the long-run growth of an independent economy, it is important that developed countries not offer stock of new technology but instead offer the capability to create new technology. These parameters might corresponds to infrastructure or institutional efficiency. Thus, above results are summarized as follows

**Proposition 3** *Interest-rate subsidy raises the welfare level through an increase in capital stock. The subsidies can increase the long-run growth rate if an optimal path of the economy is a steady growth path. The subsidy policy for interest rates enables the country to escape from poverty traps if that country has an optimal RSS path. For a country with an optimal SSS path,*

*ODA that offers endowments such as capital stock and knowledge is ineffective for the country to ride on an optimal RSS path. If the ODA sufficiently promotes production efficiency, for example, the cost of intermediate goods production  $\eta$  and R&D efficiency  $\delta$ , will change the long-run steady state of the economy to RSS.*

## 6 Conclusion

This study developed a model with capital R&D inputs and investigated the mechanics of capital on economic growth and development. The equilibrium capital stock is positively related with the long-run growth rate. The capital stock negatively depends on the cost of intermediate goods and a subjective discount rate, which determines the steady state of the economy. Each economy has a unique steady state and a unique transition path converging to the steady states. To achieve long-run positive growth, the country must have high R&D efficiency along with a low cost of intermediate goods, and a subjective discount rate. If an economy lacks these conditions, the country stays in the regime without R&D and is caught in a poverty trap.

The model presents a regime switch of the economic growth phase. If a country with positive long-run growth has a low initial capital endowment, the economy grows by capital accumulation at the first stage of economic development, and then, sufficiently accumulated capital stock enables the economy to switch to R&D-based growth, therefore, the economy realizes long-run growth through R&D.

Because the model incorporates a monopoly situation of intermediate goods production, the economy contains a distortion. For this reason, the economic policy is effective. This distortion appears in the interest rate of the economy and creates a smaller amount of long-run capital stock. Therefore, economic policies to subsidize the interest rate and increase the steady-state capital can ride the economy onto an optimal path; therefore, economic welfare can always be improved by this subsidizing policy. However, this policy cannot launch a country in a no-growth steady state under the optimal growth path onto one with long-run growth rate. Some external physical and knowledge capital aids are also ineffective for this purpose. Effective aid should improve the R&D economic environment, for example, promoting the efficiency of intermediate goods production or R&D investment.

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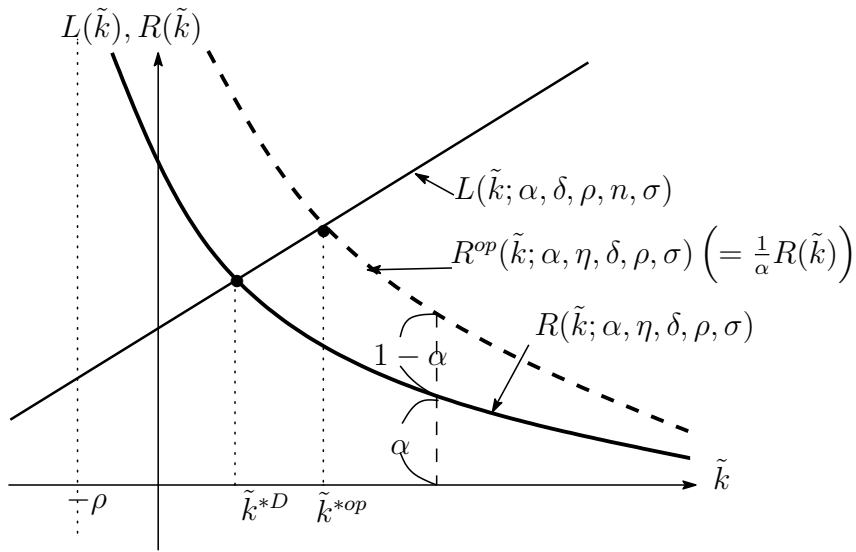


Figure 1: Equilibrium Stock of the Knowledge-adjusted Par Capita Capital in Steady Growth

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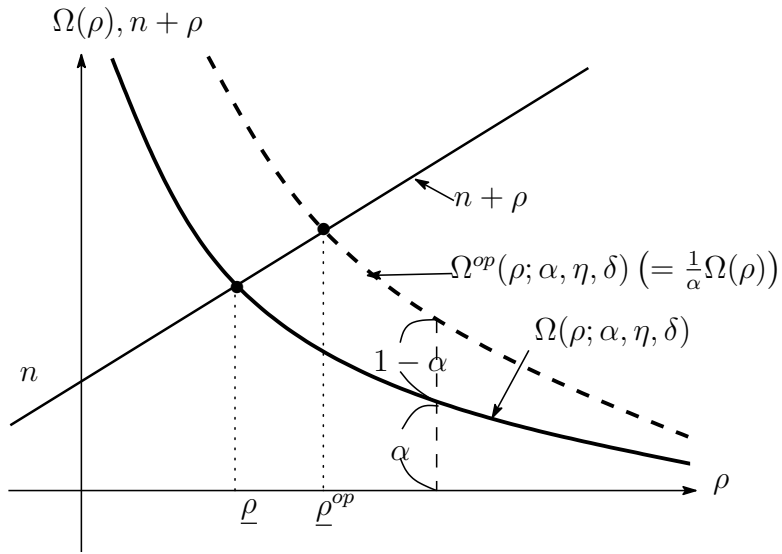


Figure 2: Upper bound for subjective discount rate for Steady Growth

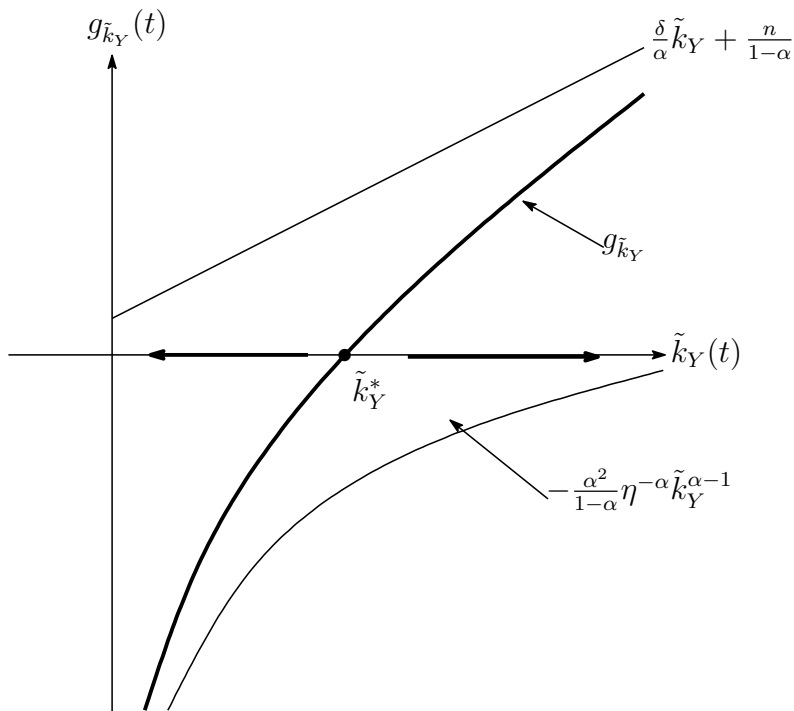


Figure 3: Dynamics of  $\tilde{k}_Y$

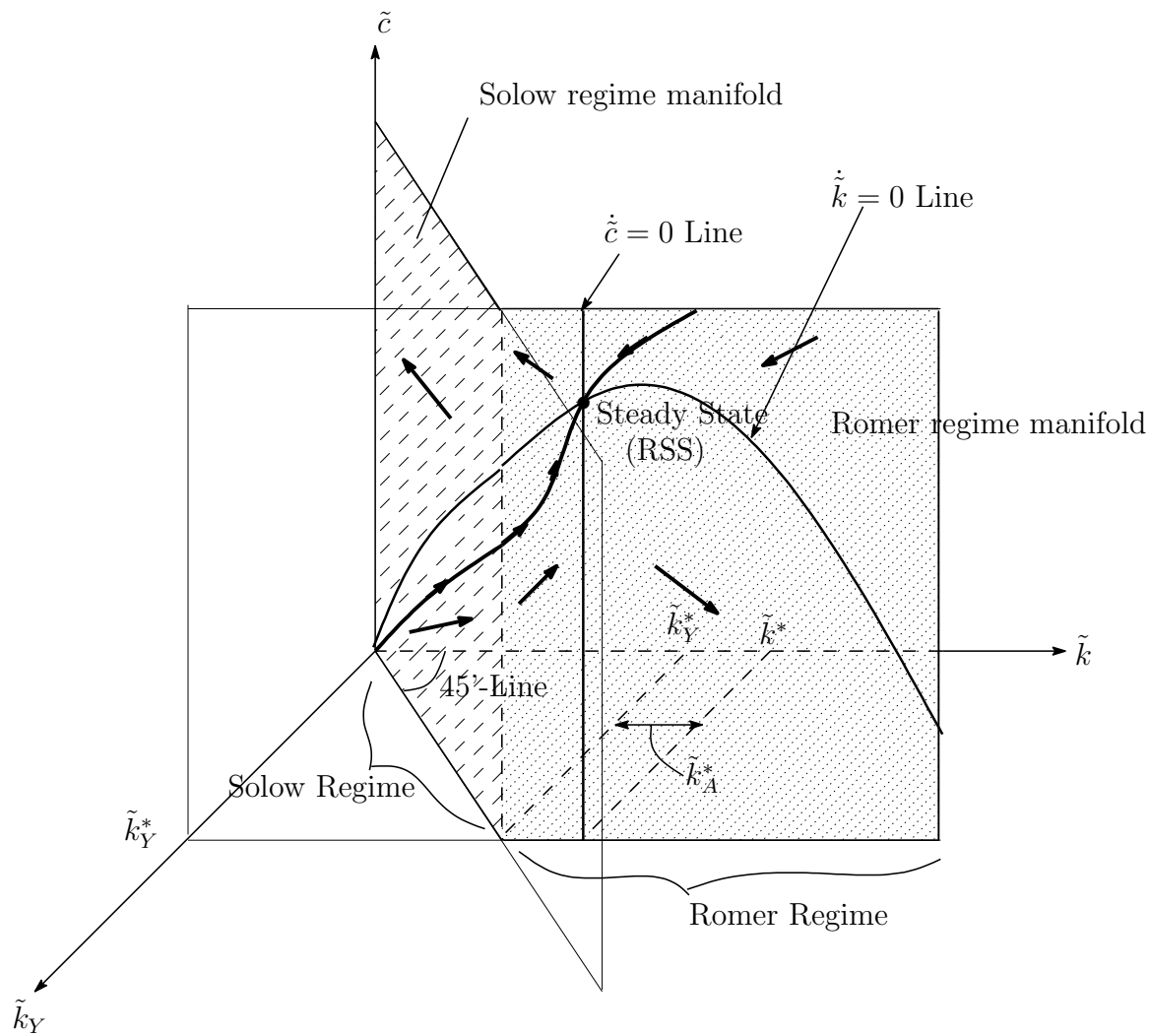


Figure 4: Global Phase Dynamics (Long-run growth case)

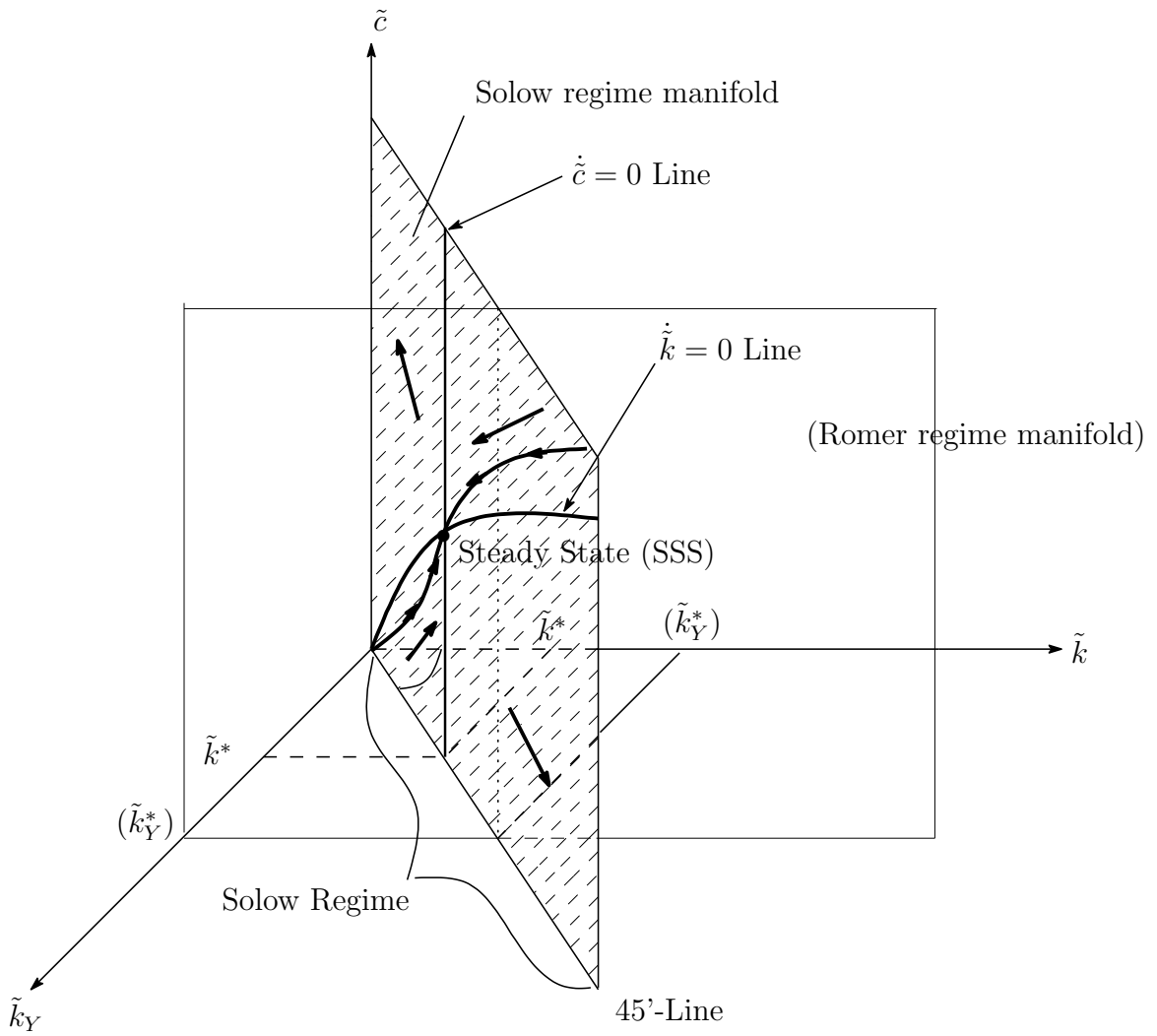


Figure 5: Global Phase Dynamics (Poverty traps case)