The Mechanics of Economic Growth through Capital Accumulation and Technological Progress*

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Abstract

This study develops a model wherein capital is used in final goods production and research and development (R&D) activities. This arrangement generates changes in the equilibrium capital allocation that are proportional to capital accumulation, which engenders a regime change from capital-based growth with decreasing returns to R&D-based perpetual growth. These two growth phases account for the polarization of economies. The model also engenders multiple equilibria on capital allocation-which emerge during the middle stages of capital accumulation-that account for leapfrogging and the instability of the economic growth of developing countries with medium capital accumulation.

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1 Introduction

This study aims to investigate the mechanics of the large diversity in economic growth rates, which remains an important subject of macroeconomic studies (see, for example, Lucas (1988)). Easterly (1994) and Quah (1996, 1997) presented evidence that as a group, the richest economies in the world are growing faster than the poorest ones. Consequently, the global income distribution suggests a polarization of world economies into two groups-the rich and the poor. In this situation, middle-class countries are also divided into rich and poor groups. Among other researchers, Krugman (1991) and Lucas (1993) observed that countries that are very similar, for example, the Philippines and South Korea, sometimes experience very different growth patterns. Moreover, the relative GDP ranking of these countries is time variant; this is referred to as "leapfrogging," as seen in Brezis, Krugman, and Tsiddon (2001). These growth patterns are illustrated in Figure 1. Thus, studies on economic growth and development must explain the following four types of economic growth patterns: (i) the continued prosperity of rich countries, (ii) the development of certain poor countries, (iii) the path followed by a country as it shifts from a medium economy to a poor one, and (iv) the continued poverty of poor countries (those caught in poverty traps).

The change in the growth engine is found in these processes of economic growth. Abramovitz and David (1973) demonstrated that at the beginning of the 19th century, America's economic growth relied more heavily on capital accumulation than on total factor productivity (TFP). Hayami and Ogasawara (1999) reported similar results from Japanese pre-war data. These works show that relatively developed economies grow as a result of capital accumulation at the early development stage. Subsequently, these countries change the growth regime to one that is driven by research and development (R&D) activities. Since capital has a decreasing returns property and R&D activities perpetually increase the TFP, this regime change appears to be a critical event for a long-run economic growth.

The purpose of this study is to elucidate the mechanics of the phenomena described above. Regime change and the realization of long-run growth have been receiving attention in this decade because of their connection with the endogenous growth theory. Some theoretical works, e.g., Zilibotti (1995), Matsuyama (1999), Galor and Moav (2004), and Irmen (2005), have developed models to describe the regime change from capital-based growth with decreasing returns to long-run positive growth. In particular, the present study is in agreement with that of Irmen (2005) in terms of the analysis on growth through capital accumulation (capital-based growth) and endogenous technological progress (R&D-based growth), and the regime change from capital-based growth to R&D-based growth. While Irmen (2005) conducted the analysis using the model of competitive economy, the present study develops a model with monopoly power and sheds light on the role of capital in economic growth phenomena such as regime change, polarization, and leapfrogging.

Our model is a modified version of the Barro and Sala-i-Martin model (1995, Ch 7), which was developed with quality improvements of intermediate goods. The striking modification of our model is that capital accumulation is introduced and capital is used in both R&D investment and intermediate goods production[1]. Most models of endogenous technological change subsume human resources (labor or human capital) that are used in both goods production and R&D activities. The amount of human resources input to the R&D sector determines the growth rate because knowledge accumulation through R&D is the engine of growth. These standard R&D-based growth studies often conclude that the introduction of capital will not alter most of the basic results, as seen in Grossman and Helpman (1991, Ch 5) and Aghion and Howitt (1998, Ch 3). In the present study, however, capital plays critical roles in the processes of economic growth and development, as described hereafter. In a country with little capital, the demand for inno-

[1]Although it is possible to refer to Shell (1967, 1973) for neoclassical growth models with capital for R&D, it is difficult to state that sufficient studies have been devoted to this subject since then.

vation is too small for R&D to be profitable. Therefore, economic growth is achieved through capital accumulation. Due to the decreasing returns of capital, this growth regime cannot establish a permanent per capita GDP growth. If capital-based economic growth allows the accumulation of sufficient capital stock for R&D activity to be profitable, then the economic regime changes to an R&D-based growth. For median amounts of capital, both positive R&D investment and the lack of it can yield an equilibrium. As a result, self-fulfilling expectations determine the realized R&D allocation.

The paper is organized as follows. The model is developed and static equilibria are solved in Section 2. The two types of steady states are derived in Section 3. In Section 4, several topics of economic growth and development are discussed. Finally, the study is concluded in Section 5.

2 The Model

2.1 Production

This study fundamentally incorporates the R&D activities for quality improvement with the multiple intermediate goods sectors - developed by Grossman and Helpman (1991, Ch 4) and Barro and Sala-i-Martin (1995, Ch 7) - into the model of Romer (1990), who originally developed the R&D-based growth model with capital accumulation. The present analysis includes three sectors: final goods, intermediate goods, and R&D. It also contains three factors: labor, capital, and knowledge. Time is discrete and extends from 0 to infinity. The price of final goods is normalized to 1.

Final goods, which can be used for consumption and capital investment, are supplied competitively and produced with labor (L) and a cluster of intermediate goods[2]. In this paper, capital can be used for final goods production (K_Y) and R&D activities (K_A). The market clearing condition for capital

[2] The quality of the cluster, that is, the productivity of intermediate goods, can be regarded as knowledge in this economy.

imposes $K = K_Y + K_A$, where K is the total amount of capital available in the economy. The labor force is employed only in the final goods sector.

The producers of final goods utilize a variety of intermediate goods and labor. Each type of intermediate good is indexed as $i \in 1, 2, ..N$, where N is assumed to be a given large constant. The final goods production function is specified as

$$Y = L^{1-\alpha} \sum_{i=1}^{N} \tilde{x}_i^{\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where Y, L, and $\tilde{x}_i$ are the output, labor, and index of the ith intermediate good, respectively. Each type of intermediate good has a vertical quality level known as a "quality ladder" along which innovations can occur. Each quality level in the ith sector is indexed as $m_i = 1, .., M_i$, where M_i is the incumbent highest quality level of the ith sector, and the quality ladder $m_i = 1, 2, .., M_i$ has the quality levels $\lambda, \lambda^2, .., \lambda^{M_i}$. Thus, the index $\tilde{x}_i$ is given as

$$\tilde{x}_i = \sum_{m=0}^{M_i} \lambda^m x_{(i,m)}, \tag{2}$$

where $x_{(i,m)}$ and $\lambda(> 1)$ are the ith intermediate good inputs with an mth quality level and the exogenously given "width" of one innovation, respectively.

In this setting, each quality level m has an efficiency level of λ^m, and therefore, the intermediate goods that are one grade higher in terms of quality are λ times more efficient than those that are one grade lower in terms of quality. Since quality-adjusted intermediate goods within the same sector are perfect substitutes, there exists a demand for quality goods with the lowest quality-adjusted cost. The first order conditions (FOCs) of production are $\frac{\partial Y}{\partial L} = w$ and $\frac{\partial Y}{\partial x_{(i,m)}} = p_{(i,m)}$ for $x_{(i,m)} > 0$, where w and $p_{(i,m)}$ are the real wage and the price of the ith intermediate good of the mth quality, respectively.

R&D firms facilitate technological progress; they create a design that is one grade higher in terms of quality than a design that has the incumbent highest quality level. The R&D activities of the firms are overtaken at the beginning of each period, and the results are immediately evident. A successful research firm retains exclusive rights to the use of intermediate goods of this new quality level for one period. This exclusive right is referred to as a "patent." If multiple firms succeed simultaneously, the patent is randomly given to one firm. The one-period patent generates two economic effects. First, the monopoly enjoyed by the innovator provides an incentive for innovation. Second, when all the R&D activities of a sector fail to generate a new quality level, all the patents of that sector lapse and the intermediate goods required by that sector can be supplied competitively. Thus, the price of the ith intermediate good is monopolistic only if the R&D activity in the ith sector is successful. Consequently, such a sector is known as a "monopolized sector." However, if the R&D activity in the i'th sector fails, then the i'th intermediate good can be supplied competitively. Such a sector is known as a "competitive sector." The sets of the competitive and monopolized sectors are given as

$$\begin{aligned}\mathcal{C} &= \{i \in \{1,..,N\} \,|\text{Sector } i \text{ is competitive.}\} \\ \mathcal{M} &= \{i \in \{1,..,N\} \,|\text{Sector } i \text{ is monopolized.}\}\end{aligned}$$

In this study, the intermediate goods production follows Romer (1990). These goods are used only in the final goods production process and are produced by using capital. It is assumed that one unit of intermediate goods is generated by η units of capital and the rental price of capital is r. Hence, the firm producing the ith intermediate good maximizes the profit such that $p_{(i,m)}x_{(i,m)} - r\,\eta\,x_{(i,m)}$.

In the competitive sector, the patent term of the top quality expires, and therefore, all firms can operate freely with top quality technology M_i. The competitive supply of intermediate goods equates the price to the marginal cost. As a result, the following conditions are obtained:

$$\text{For} \quad i \in \mathcal{C} \;\Rightarrow\; \quad x_{(i,M_i)} = \left(\frac{\alpha}{r\eta}\right)^{\frac{1}{1-\alpha}} L\lambda^{\frac{\alpha}{1-\alpha}M_i}, \text{ and } \quad p_{(i,M_i)} = r\eta. \qquad (3)$$

In the monopolized sector, a monopoly firm-the firm that holds a patent-maximizes its profit by considering price as a control variable. Therefore, the FOC of the monopoly firm in the ith sector with the M_ith quality yields the following:

$$\text{For} \quad i \in \mathcal{M} \Rightarrow \quad x_{(i,M_i)} = \left(\frac{\alpha^2}{r\,\eta}\right)^{\frac{1}{1-\alpha}} L\lambda^{\frac{\alpha}{1-\alpha}M_i}, \text{and} \quad p_{(i,M_i)} = \frac{r\,\eta}{\alpha}. \tag{4}$$

With regard to monopolistic pricing, we have the following three conditions. First, each unit of the top quality is equivalent to $\lambda > 1$ unit of a good with the next best quality. Second, a good that is one grade lower than the top quality good is supplied at marginal cost $r\eta$ because the patent for this grade expires. Third, the different quality grades are perfect substitutes if they are weighted by the quality level. Based on the above conditions, it follows that $p_{(i,M_i)} < \lambda r\eta$ is necessary for the firm innovating a top quality good to monopolize the demand of that good. Therefore, combining (4) and $p_{(i,M_i)} < \lambda r\eta$ shows that the condition of (4) is optimal under the assumption that $1/\alpha < \lambda$. The following discussion is developed such that it satisfies the present assumption[3]. Thus, only top quality goods are supplied in competitive and monopolized sectors.

The aggregate indices of intermediate goods and quality are defined as

$$\sum_{i=1}^{N} x_{(i,M_i)} \equiv X \equiv \frac{1}{\eta}K_Y, \quad \sum_{i\in\mathcal{C}} \lambda^{\frac{\alpha}{1-\alpha}M_i} \equiv Q_{\mathcal{C}}, \quad \text{and} \quad \sum_{i\in\mathcal{M}} \lambda^{\frac{\alpha}{1-\alpha}M_i} \equiv Q_{\mathcal{M}}, \tag{5}$$

where $X = \frac{1}{\eta}K_A$ stems from the assumption that η unit of capital is transformed into one unit of the intermediate good. It should be noted that $Q \equiv \sum_i \lambda^{\frac{\alpha}{1-\alpha}M_i} = Q_{\mathcal{C}} + Q_{\mathcal{M}}$ holds.

Using the optimal conditions for final and intermediate goods production, we can obtain the following variables: output, interest rate, wage rate, and profit of intermediate goods firms (see Appendix A for more detailed derivations). Output is derived as

$$Y = \eta^{-\alpha} L^{1-\alpha} K_Y^{\alpha}\, \Psi(\mathcal{M}, \alpha, Q), \tag{6}$$

[3]This assumption implies that the width of one innovation is sufficiently large. If $1/\alpha < \lambda$, then the optimal pricing is given as $p_{(i,M_i)} = \lambda r\eta$. This pricing does not alter the main framework of the model; therefore, we assume that $1/\alpha < \lambda$ throughout this study.

where

$$\Psi(\mathcal{M},\alpha,Q) \equiv \frac{Q - Q_{\mathcal{M}} + \alpha^{\frac{\alpha}{1-\alpha}} Q_{\mathcal{M}}}{(Q - Q_{\mathcal{M}} + \alpha^{\frac{1}{1-\alpha}} Q_{\mathcal{M}})^{\alpha}}.$$

Function Ψ captures the GDP level that is affected by the degree of monopoly and the quality index. For descriptive convenience, we introduce a new variable:

$$\Phi(\mathcal{M},\chi,Q) \equiv Q_{\mathcal{C}} + \alpha^{\chi} Q_{\mathcal{M}} = Q - Q_{\mathcal{M}} + \alpha^{\chi} Q_{\mathcal{M}},$$

where $\chi = \{\frac{1}{1-\alpha}, \frac{\alpha}{1-\alpha}\}$. Using function Φ, we obtain the following relationship:

$$\Psi(\mathcal{M},\alpha,Q) = \begin{cases} Q^{1-\alpha} & \text{if} \quad \mathcal{C} \text{ or } \mathcal{M} = \{\phi\} \\ \frac{\Phi\left(\mathcal{M},\frac{\alpha}{1-\alpha},Q\right)}{\Phi\left(\mathcal{M},\frac{1}{1-\alpha},Q\right)^{\alpha}}, & \text{otherwise} \end{cases}. \tag{7}$$

Using expressions Y and Φ, r and w can be written as

$$w = (1-\alpha)\frac{Y}{L}, \quad \text{and} \quad r = \alpha \frac{\Phi(\mathcal{M}, \frac{1}{1-\alpha}, Q)}{\Phi(\mathcal{M}, \frac{\alpha}{1-\alpha}, Q)} \frac{Y}{K_Y}. \tag{8}$$

The profit of the ith sector monopoly firm that has a patent of quality M_i is given as

$$\pi_{(i,M_i)} = \varphi \frac{Y}{\Phi\left(\mathcal{M}, \frac{\alpha}{1-\alpha}, Q\right)} \lambda^{\frac{\alpha}{1-\alpha} M_i}, \tag{9}$$

where $\varphi \equiv (1-\alpha)\alpha^{\frac{1}{1-\alpha}}$.

2.2 R&D Activities

It is presumed that R&D activities for the purpose of innovating different quality levels are conducted using capital, and the success of R&D stochastically depends on the input capital. In this study, we assume the following properties of probabilities: the innovation probability is the same across sectors, and probability μ presumably depends on the capital equipment rate for R&D activities that are defined by $k_A \equiv K_A/L$; therefore, probability can be denoted as $\mu = \mu(k_A)$. Further, it is assumed that innovation does not occur if the R&D activity is not undertaken; therefore, $\mu(0) = 0$. When innovation occurs in a sector, the probability that firm j in that sector will be granted a patent is assumed to be proportional to the share of R&D input in the ith sector; therefore, K^j_{Ai}/K_{Ai}, where K^j_{Ai} and K_{Ai} represent the R&D input for the ith sector of firm j and the aggregate R&D input for the ith sector, respectively. Since the patent term is one period, the value of an innovation equals the monopoly of the profits of the intermediate good sector $\pi_{(i,M_i)}$.

Therefore, from the above assumptions, it is shown that the profit of sector i of R&D firm j is

$$\max_{K^j_{Ai}} \quad \frac{\mu K^j_{Ai}}{K_{Ai}} \pi_{(i,M_i)} - r\, K^j_{Ai}.$$

The presence or absence of capital investment for R&D activities is determined as follows. The allocation of capital for goods production and R&D activities is adjusted through rental price r. (8) implies that a shift of K_Y to 0 can change rental price r to infinity. Therefore, capital is always used in goods production. In contrast, as explained below, capital is not always allocated for R&D activities. Since it is assumed that the R&D sector can be entered into freely, zero profit must hold in the equilibrium. If $\frac{\mu K^j_{Ai}}{K_{Ai}} \pi_{(i,M_i)} > r\, K^j_{Ai}$, then an infinite amount of capital would be used as input for R&D: this cannot pertain to an equilibrium. If $\frac{\mu K^j_{Ai}}{K_{Ai}} \pi_{(i,M_i)} < r\, K^j_{Ai}$ holds, then R&D investment is less profitable than the input to produce intermediate goods. Consequently, the R&D input stops and equilibrium is attained without R&D; therefore the probability of R&D success is 0, that is, $\mu = 0$. If $\mu = 0$ is realized, then the quality of intermediate goods would remain constant over time. If $\frac{\mu K^j_{Ai}}{K_{Ai}} \pi_{(i,M_i)} = r\, K^j_{Ai}$, then a positive amount of capital would be devoted to R&D activities and the market would be in equilibrium. The above points can be summarized as follows:

$$\frac{\mu}{K_{Ai}} \pi_{(i,M_i)} \leq r. \quad \text{with equality whenever } K_{Ai} > 0. \tag{10}$$

If $K_{Ai} > 0$, then (10) necessarily holds with equality. Substituting (8) and (9) into (10), we obtain

$$\frac{\mu}{K_{Ai}} \varphi \frac{Y}{\Phi\left(\mathcal{M}, \frac{\alpha}{1-\alpha}, Q\right)} \lambda^{\frac{\alpha}{1-\alpha} M_i} \leq \alpha \frac{\Phi\left(\mathcal{M}, \frac{1}{1-\alpha}, Q\right)}{\Phi\left(\mathcal{M}, \frac{\alpha}{1-\alpha}, Q\right)} \frac{Y}{K_Y}. \tag{11}$$

For positive K_{Ai}, (11) is transformed into

$$K_{Ai} = \nu \lambda^{\frac{\alpha}{1-\alpha} M_i} \quad \text{where} \quad \nu \equiv \frac{\mu \varphi K_Y}{\alpha \Phi\left(\mathcal{M}, \frac{\alpha}{1-\alpha}, Q\right)}.$$

Aggregating the above equation with i, we obtain the following relationship $K_A = \nu Q$. Combining $K_A = \nu Q$ and $K_{Ai} = \nu \lambda^{\frac{\alpha}{1-\alpha} M_i}$ yields $\lambda^{\frac{\alpha}{1-\alpha} M_i}/K_{Ai} = Q/K_A$. Substituting this relationship into (11), we obtain the condition of R&D as follows:

$$\mu\, \varphi \frac{Q}{K_A} \leq \frac{\alpha \Phi(\mathcal{M}, \frac{1}{1-\alpha}, Q)}{K_Y}, \quad \text{with equality whenever } K_A > 0. \qquad (R)$$

The number of sectors N is assumed to be sufficiently large, such that the average quality in competitive and monopolized sectors becomes identical. Consequently, the relationships between R&D investment K_A and the quality indices are derived as follows:

$$Q_{\mathcal{M}} = \mu Q \qquad \text{and} \qquad Q_{\mathcal{C}} = (1-\mu) Q. \qquad (12)$$

From (12), both sides of the above equation of Condition (R), when divided by the per capita quality indices Q/L, become

$$\begin{aligned} LHS(k_A) &= \frac{\mu(k_A)\varphi}{k_A}, \\ RHS(k_A) &= \frac{\alpha\left(1 - \mu(k_A) + \alpha^{\frac{1}{1-\alpha}} \mu(k_A)\right)}{k - k_A}, \end{aligned}$$

where $k \equiv K/L$ denotes per capita capital stock in the economy. In the above equations and henceforth in this paper, LHS refers to the left-hand side and represents the marginal value of R&D investment; RHS implies the right-hand side and shows the cost of R&D. These two equations determine the equilibrium capital allocations.

2.3 Equilibrium Capital Allocations

In this section, the arbitrage condition of capital allocation is solved and equilibrium capital allocations are derived. To solve the arbitrage condition, it is necessary to specify the probability function μ; this is done as follows:

$$\mu(k_A) = \begin{cases} \delta k_A & \text{for} \quad 0 < k_A < \frac{1}{\delta} \\ 1 & \text{for} \quad k_A > \frac{1}{\delta} \end{cases} . \tag{P}$$

This function increases linearly when $0 < k_A < \frac{1}{\delta}$ and is constant to 1 when $k_A \geq \frac{1}{\delta}$. This form may appear to be very specific. However, it holds the usual properties of probability, such as

$$0 \leq \mu(k_A) \leq 1, \ \mu(0) = 0, \ \lim_{k_A \to \infty} \mu(k_A) = 1, \text{ and } \mu'(k_A) \geq 0. \tag{13}$$

Further, all the obtained types of equilibrium capital allocations are obtainable as similar forms in the more general specification of assumptions[4]. In addition, this specification analytically provides a solution for (R), which simplifies the analysis. Therefore, this basic functional form is adopted in the following part of this study. From (12), (7) is translated into $\psi(k_A)Q^{1-\alpha}$, where

$$\psi(k_A) = \frac{1 - \mu(k_A) + \alpha^{\frac{\alpha}{1-\alpha}} \mu(k_A)}{\left(1 - \mu(k_A) + \alpha^{\frac{1}{1-\alpha}} \mu(k_A)\right)^{\alpha}}, \quad \text{and} \quad \psi(1) = \psi(0) = 1. \tag{14}$$

From (R) and (P), the equilibrium of capital allocation for R&D activities, which we term as "equilibrium capital allocation" (or "equilibrium"), is obtained as follows:

$$\begin{array}{rc} \text{An equilibrium without R\&D exists, if} & k \leq k^+, \\ \text{An equilibrium with Low R\&D exists, if} & k^- \leq k \leq k^+, \\ \text{An equilibrium with High R\&D exists, if} & k^- \leq k, \end{array} \tag{A}$$

where $k^- \equiv \frac{1}{\delta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}$ and $k^+ \equiv \frac{1}{\delta(1-\alpha)}$. We denote the equilibrium capital allocation for R&D activities in each case by k_A^0, $\bar{k}_A$ and k_A^*, respectively, that is, $k_A^0 \equiv 0$, $\bar{k}_A \equiv \frac{1-\delta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} k}{\delta(1-\alpha^{\frac{\alpha}{1-\alpha}})}$, $k_A^* \equiv (1-\alpha)k$. The order of these values is given by the proposition in Appendix B. Figure 2 (a)-(c) depict the curves

[4] For example, a smooth function form satisfying (13) will generate similar types of capital allocations obtained from assumption (P). Moreover, because assumption (P) can provide analytical solutions, it is adopted for simplicity.

of the RHS and LHS and the equilibrium capital allocations for each capital stock.

The appearance of these equilibrium capital allocations is conditioned by the amount of aggregate capital stock. Therefore, these allocations are divided into the following three "equilibrium sets" (or "set") that correspond with the amount of capital accumulation. These sets $\mathcal{K}_A$ are obtained as follows:

$$
\begin{array}{llll}
\text{Set of an equilibrium without R\&D:} & \mathcal{K}_A^N := \{k_A^0\} & \text{if} & k \leq k^-, \\
\text{Set of multiple equilibria:} & \mathcal{K}_A^M := \{k_A^0, \bar{k}_A, k_A^*\} & \text{if} & k^- \leq k \leq k^+, \\
\text{Set of an aquilibrium with large R\&D:} & \mathcal{K}_A^L := \{k_A^*\} & \text{if} & k^+ \leq k.
\end{array}
$$

See Appendices B and C for detailed derivation of these equilibria and sets. The equilibrium sets are classified by the following three stages of capital stock: $k < k^-$, $k^- < k < k^+$, and $k^+ < k$. We term these stages as low capital, middle capital, and large capital, respectively. Table 1 lists these equilibria. With regard to these equilibria, it should be noted that by definition, k_A^0 is related to $\mu(k_A^0) = 0$ and k_A^* is always related to $\mu(k_A^*) = 1$ because $k_A = (1-\alpha)k$ and $k > k^- = \frac{1}{\delta(1-\alpha)}$, implying that $k_A > (1/\delta)$.

3 Dynamical System and Steady States

3.1 Dynamical System

Apart from the quality improvements described in the previous section, two additional dynamics of growth factors are introduced in the model. The first is capital accumulation through a savings mechanism and the second is exogenous growth of the labor supply. The first is capital accumulation through a savings mechanism and the second is exogenous population growth. We assume a simple overlapping generations (OLG) model as the savings mechanism. An individual born in period t is defined as a member of the tth generation; this individual is assumed to live in two periods and have the utility function $U_t = \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1}$. Representative individuals of each generation inelastically supply one unit of labor during the period of their youth and then retire during the period of their old age. Therefore, the young save for the period of retirement and the old withdraw their saving. This generates an asset market. A budget constraint of the tth generation is given as $c_{1t} + \frac{1}{1+r_{t+1}} c_{2t+1} = w_t$. The optimization of consumption derives the capital dynamics as follows:

$$K_{t+1} = \frac{w_t}{2+\rho} L_t = \frac{1-\alpha}{2+\rho} \psi(k_{At}) \eta^{-\alpha} L_t^{1-\alpha} K_{Yt}^{\alpha} Q_t^{1-\alpha} . \qquad (K)$$

The second dynamics of growth factors is labor supply. The present model assumes that population grows at an exogenously given constant rate n. An individual born at time t inelastically supplies one unit of labor, and generation t represents the unit to which the individual born at time t belongs. Therefore, the aggregate labor supply at time t can be identified with the population size of the tth generation, which can be given as

$$L_t = (1+n)^t L_0, \qquad (t = 0, 1, 2, ...) \qquad (L)$$

where L_0 is the initial young population.

The setup of R&D yields the quality index growth in the following manner:

$$E\left(\frac{\Delta Q}{Q}\right) = \mu(k_A)(\lambda^{\frac{\alpha}{1-\alpha}} - 1).$$

Since there are sufficiently numerous N sectors, the above equation can equivalently be rewritten as

$$Q_{t+1} = \left\{1 + \mu(k_A)(\lambda^{\frac{\alpha}{1-\alpha}} - 1)\right\} Q_t. \qquad (Q)$$

In addition, if $\mu(k_A) = 1$, that is, $k_A \geq (1/\delta)$, then the equation becomes $Q_{t+1} = \lambda^{\frac{\alpha}{1-\alpha}} Q_t$; if $\mu(k_A) = 0$, that is, $k_A = 0$, then the equation becomes $Q_t = Q_C = \bar{Q}$.

(K), (L), and (Q) constitute the dynamical system of the economy. To obtain the dynamics from k_t to k_{t+1}, (K) is rewritten as

$$k_{t+1} = \frac{1-\alpha}{(1+n)(2+\rho)} \psi((1-s_t)k_t) \eta^{-\alpha} s_t^{\alpha} k_t^{\alpha} Q_t^{1-\alpha}, \qquad (15)$$

where $k_Y = sk$, $k_A = (1-s)k$, and

$$s_t \equiv \begin{cases} 1 & \text{if} \quad k_A = k_A^0 \\ \frac{1}{\delta(1-\alpha^{\frac{\alpha}{1-\alpha}})k_t} & \text{if} \quad k_A = \bar{k}_A \\ \alpha & \text{if} \quad k_A = k_A^* \end{cases}.$$

It is useful to express the dynamical system in terms of variables that will remain constant in the steady state. Hence, a quality-adjusted per capita capital stock $\tilde{k}_t \equiv K_t/(L_t Q_t)$ or k_t/Q_t is introduced. From (Q), (15) and this new variable, we obtain the dynamics of capital $\tilde{k}$ as follows:

$$\tilde{k}_{t+1} = \frac{(1-\alpha)\psi((1-s_t)k_t)\eta^{-\alpha}s_t^{\alpha}}{\left\{1+\tilde{\mu}((1-s_t)k_t)(\lambda^{\frac{\alpha}{1-\alpha}}-1)\right\}(2+\rho)(1+n)}\tilde{k}_t^{\alpha}(\equiv(\phi(\tilde{k}_t;k_A)). \quad (16)$$

3.2 The Solow Regime

Quality does not increase when an economy is in equilibrium without R&D. In this case, the system cannot grow through R&D but it can grow through capital accumulation. Therefore, the growth in this regime corresponds to a neoclassical growth model without technological change. Following Matsuyama (1999), we term this regime as the Solow regime. The capital dynamics (15) in this case become

$$\tilde{k}_{t+1} = \frac{(1-\alpha)\eta^{-\alpha}}{(2+\rho)(1+n)}\tilde{k}_t^{\alpha}(=\phi(\tilde{k};k_A^0)) \quad \text{or} \quad k_{t+1} = \frac{(1-\alpha)\eta^{-\alpha}}{(2+\rho)(1+n)}k_t^{\alpha}\bar{Q}^{1-\alpha},$$

where z represents the per capita value of Z: $k_t = K_t/L_t$. The above equation has a stable equilibrium that converges to the (per capita) steady-state capital stock in the Solow regime k^{S*}, which is given as follows:

$$\tilde{k}^{S*} = \left(\frac{1-\alpha}{2+\rho}\eta^{-\alpha}\right)^{\frac{1}{1-\alpha}}, \quad \text{or} \quad k^{S*} = \left(\frac{1-\alpha}{2+\rho}\eta^{-\alpha}\right)^{\frac{1}{1-\alpha}}\bar{Q}. \quad (17)$$

By definition, the Solow regime is that in which the economy is in equilibrium without R&D ($k_A = 0$). From (A), this is equivalent to holding $k < K^+$. From (A) and (17), the steady state of the Solow Regime k^{S*} must satify the following condition:

$$k^{*S} < k^+ \Longrightarrow \bar{Q} < Q^+\left(\equiv \frac{1}{\delta(1-\alpha)}\left(\frac{(2+\rho)(1+n)\eta^{\alpha}}{(1-\alpha)^{2-\alpha}\alpha^{\alpha}}\right)^{\frac{1}{1-\alpha}}\right).$$

3.3 The Romer Regime

Quality increases when an economy is in equilibrium with positive R&D. In this case, the system grows through R&D activities in addition to capital accumulation. Therefore, the growth in this regime corresponds to an endogenous growth model with technological progress. Following Matsuyama (1999), we term this regime as the Romer regime.

An economy that is endowed with capital more than k^- has the equilibria of $\bar{k}_A$ and k_A^*. Therefore, this economy has the capability of an R&D-based growth. For positive R&D activities, an equilibrium R&D investment must include $\{\bar{k}_A, k_A^*\}$. However, $\bar{k}_A$ can be neither a long-run solution for steady growth nor a periodic solution set. This is because the following mechanism is involved in positive R&D activities. Since s_t is the function of k_t and k_t increases with an increase in Q_t, even if $\tilde{k}$ provides the steady state of (16), then the value of this steady state is shifted by a change in k_t. Further, k^+, the upper limit of $\bar{k}_A$, is violated by the growth of k_A, which is encouraged by the growth of Q through (15). With regard to k_A^*, this equilibrium value leads to $s = \alpha$ and $\mu(k_A^*) = 1$, and therefore, the dynamics of $\tilde{k}_t$ are independent of k_t. As a result, the k_A^* is the only equilibrium that comprises the steady state for the Romer regime.

Thus, the dynamic equation in this regime is transformed into the following:

$$\tilde{k}_{t+1} = \frac{(1-\alpha)\eta^{-\alpha}\alpha^{\alpha}}{\lambda^{\frac{\alpha}{1-\alpha}}(2+\rho)(1+n)}\tilde{k}_t^{\alpha}(=\phi(\tilde{k}_t; k_A^*));$$

$\tilde{k}_t$ converges to a steady state, which is written as

$$\tilde{k}^{R*} = \left(\frac{(1-\alpha)\eta^{-\alpha}\alpha^{\alpha}}{\lambda^{\frac{\alpha}{1-\alpha}}(2+\rho)(1+n)}\right)^{\frac{1}{1-\alpha}}. \tag{18}$$

For this $\tilde{k}^{R*}$ to be the equilibrium of the Romer regime, $\tilde{k}^{R*} > \tilde{k}^-$ (from (A)) must hold. Combining (A) and (18), we obtain the following condition:

$$\eta^{\alpha}\lambda^{\frac{\alpha}{1-\alpha}}\delta^{\frac{-1}{1-\alpha}}(2+\rho)(1+n) < \alpha^{2\alpha}(1-\alpha)^{\frac{2\alpha}{1-\alpha}}.$$

This inequality implies that higher δ and lower n, λ, ρ, and η realize the Romer regime.

Since $\tilde{k}$ is constant in the steady state, the GDP growth rate is given as $\frac{\Delta k}{k} = \frac{\Delta Q}{Q} = \lambda^{\frac{\alpha}{1-\alpha}}$. The growth rate positively depends on the width of one innovation and not on the allocation of the resources used in R&D activities. The growth rate positively depends on the width of one innovation and not on the allocation of the resources used in R&D activities. Therefore, in the terminology used by Jones (1995), the model displays a semi-endogenous growth[5]. Although some semi-endogenous growth models show that the long-run positive growth rate is necessary for a positive population growth, economies in the present model can experience growth without population growth.

4 Growth Patterns

4.1 Polarization

The model considers two types of steady states, namely, the steadily positive growth (the steady state of the Romer regime) and the stationary state without per capita GDP growth (the steady state of the Solow regime). These steady states correspond to the paths drawn in Figure 1 – (i) the continued prosperity of rich countries and (iv) the development of certain poor countries, respectively. Therefore, the model includes the polarization of economies.

A country with a large capital has an equilibrium that comprises extensive R&D; hence, the economy achieves perpetual growth due to R&D activities. In contrast, a country with a low capital has an equilibrium without R&D, and therefore, its economy grows through capital accumulation. Further, if this country has a sufficiently low quality level, the per capita GDP growth will be suspended in the long run because of the decreasing marginal product of the capital; further, the equilibrium that is not based on R&D becomes a

[5] However, this semi-endogenousity depends on assumption (P), and therefore, it is not an inevitable property of this model. Indeed, some assumptions can connect a long-run positive growth rate with the R&D-input allocation rate.

long-run steady state.

There exist two types of poverty traps, and they have the following different policy implications. In an economy where $k^{*S} < k^{+}$, growth is suspended if the economy continues to select equilibrium $k_A = 0$. If an economy caught in a poverty trap maintains $k^{-} < k^{*S} < k^{+}$, then it has the potential to initiate profitable R&D activities. Therefore, in order to direct the economy toward R&D-based growth, the government only has to raise expectations from $k_A = 0$ to $k_A > 0$. When $k^{*S} < k^{-}(< k+)$, the economy has an equilibrium based only on R&D that does not yield profits. Hence, certain economic policies are effective in enabling countries to escape from poverty traps. This paper takes taxes and subsidies into consideration. It is proposed that if a constant rate tax $\tau > 0$ (a subsidy if $\tau < 0$) is levied (provided) on the interest (the rental price of capital) and the profit of the intermediate sector, then

$$r^{\tau} = (1 - \tau_r)r, \quad \text{and} \quad \pi^{\tau} = (1 - \tau_{\pi})\pi^{M},$$

where τ_r and τ_π are the tax (or subsidization) rates on the interest and profit, respectively. It is assumed that the government provides these tax revenues to the public through a lump-sum transfer (or finances these subsidies by levying a lump-sum tax) and by maintaining a balanced financial policy. Then, after taxation (or subsidization), the LHS and RHS are modified as follows:

$$\begin{aligned} LHS^{T} &= (1 - \tau_{\pi})\frac{\mu\varphi}{k_A}, \\ RHS^{T} &= (1 - \tau_r)\frac{\alpha(1 - \mu + \alpha^{\frac{1}{1-\alpha}}\mu)}{k - k_A}, \end{aligned}$$

where the index T indicates that taxation (or subsidization) has been effected. These equations imply that a subsidy for the profit and a tax on the interest rate are effective in enabling economies to escape from poverty traps.

4.2 Regime Change

Since the economic growth presented in this study changes the growth engine from capital accumulation to R&D, the regime change observed by Abramovitz and David (1973) and Hayami and Ogasawara (1999) is also obtained in this study. An economy with a sufficiently small initial capital endowment grows by means of capital accumulation, and after a sufficiently large capital is accumulated, the R&D investment becomes profitable. In other words, an equilibrium with a positive R&D (potentially) emerges. If R&D activities are, in fact, executed, then the Romer regime replaces the Solow regime. The dynamics of the regime change are illustrated in Figure 3. The growth of an economy endowed with a small capital follows the dynamical equation $\tilde{k}_{t+1} = \phi(\tilde{k}_t, 0)$ until the economy accumulates capital $\tilde{k}_t > \tilde{k}^-$ (Figure 3(a)). When $\tilde{k}^- < \tilde{k} < \tilde{k}^+$, the economy is capable of initiating R&D, and therefore, it may change its regime. Thus, this period is referred to as the "transition period." If R&D is not executed, then the quality index Q is fixed, and therefore, the lines of critical capital stock $\tilde{k}^+$ and $\tilde{k}^-$ are also fixed. Thus, $\tilde{k}^{S*}$ maintains the condition for a steady state. In contrast, if R&D is executed, then both $\tilde{k}^-$ and $\tilde{k}^+$ are shifted to the left. After sufficient R&D is executed, the growth of the economy follows the dynamical system $\tilde{k}_{t+1} = \phi(\tilde{k}_t, k_A^*)$ (Figure 3(b)), and the economy converges to the steady state $\tilde{k}^{R*}$ under the Romer regime. Thus, our model includes the regime change.

4.3 Diverse Growth Patterns

An economy endowed with medium capital ($\tilde{k} \in (\tilde{k}^-, \tilde{k}^+)$) is characterized by multiple equilibria of capital allocation, that is, capital allocation between production and R&D activities. This characteristic generates diverse economic growth patterns that are paths, such as (ii) the development of certain poor countries and (iii) the path followed by a country as it shifts from a medium economy to a poor one, as shown in Figure 1.

As discussed by Krugman (1991) and Lucas (1993), two similar countries may display very different growth patterns: a country that expects an equilibrium without R&D realizes such an equilibrium and the economy converges to a no-growth steady state. On the other hand, a country that expects an equilibrium with positive R&D realizes a positive R&D as well as economic growth with technological progress. Further, since the economies whose capital is within this range may experience both R&D-based growth and a no-growth steady state, there may be a change in their GDP ranking. This is the process of leapfrogging discussed in this study.

An economy with multiple equilibria includes an equilibrium without R&D. Therefore, even if the economy experiences steady growth, there always remains the possibility that the economic equilibrium may switch to one without R&D. Newly industrializing economies are sometimes hit by economic crises such as the Latin debt crisis of the 1980s and the Asian currency crisis of 1997. Newly industrializing economies are sometimes hit by economic crises such as the Latin debt crisis of the 1980s and the Asian currency crisis of 1997. Many of these crises appear to be caused due to monetary reasons or a lax financial policy; however, this study indicates that medium economies may have multiple equilibria, and therefore, these crises could be caused by the real and expectational shocks that result from jumps in the equilibria. Since one-shot innovation or an exogenous improvement in quality leads to changes in the equilibrium conditions $\tilde{k}^{-}$, $\tilde{k}^{+}$ and $\tilde{k}^{S*}$, they may exert a considerable influence on transitional and long-run growth experiences, thereby initiating industrialization.

5 Concluding Remarks

This paper presents an approach that encompasses the following important phenomena in economic growth and development: polarization, regime change, and leapfrogging. The model presented in this paper builds on the presumption that capital is an input to R&D. This modification demonstrates that the R&D activity level can be assigned to capital endowment.

The obtained equilibria are a unique equilibrium without R&D, a unique equilibrium with positive R&D, and multiple equilibria with and without R&D. The model demonstrates that as an economy accumulates capital, it changes its growth regime from one without R&D to that with positive R&D. Since R&D activities increase TFP, positive R&D enables long-run per capita GDP growth. In contrast, an equilibrium without R&D can be a long-run steady state with a small endowment of capital and quality. Therefore, the model contains two types of steady states and explains the polarization of economies.

The model includes multiple equilibria that emerge for the median levels of capital accumulation and comprise equilibria with and without R&D. These equilibria explain other phenomena of economic growth and development. An economy can jump from one equilibrium condition to another merely by changed expectations, generating leapfrogging of GDP rankings. These equilibria also account for the possibility that countries with identical economic parameters, if they form different expectations for alternative destinies of poverty traps or steady R&D-based growth, can have very different growth experiences.

Some relevant issues remain unexplored. For example, our study does not explicitly incorporate human capital. In this study, capital is regarded as a composite that includes physical and human capital components. Nevertheless, it is important to consider the inherent properties of human capital accumulation and their effects on economic growth and development.

Appendix

A. Derivation of Y, r and $\pi_{(i,M_i)}$

The aggregate GDP level Y and interest rate r are derived in Appendix A.

Substituting (2), (3), and (4) into (1) and using the quality indices $Q_{\mathcal{C}} =$

$Q - Q_{\mathcal{M}}$ and the definition of $Q_{\mathcal{M}}$ (given in (5)), we obtain

$$\begin{aligned} Y &= L^{1-\alpha}\left(\sum_{i\in\mathcal{C}}(\lambda^{M_i}x_i)^{\alpha} + \sum_{i\in\mathcal{M}}(\lambda^{M_i}x_i)^{\alpha}\right). \\ &= \left(\frac{\alpha}{r\eta}\right)^{\frac{\alpha}{1-\alpha}} L\left(\sum_{i\in\mathcal{C}}\lambda^{\frac{\alpha}{1-\alpha}M_i} + \alpha^{\frac{\alpha}{1-\alpha}}\sum_{i\in\mathcal{M}}\lambda^{\frac{\alpha}{1-\alpha}M_i}\right) \\ &= \left(\frac{\alpha}{r\eta}\right)^{\frac{\alpha}{1-\alpha}} L\,(Q - Q_{\mathcal{M}} + \alpha^{\frac{\alpha}{1-\alpha}}Q_{\mathcal{M}}). \end{aligned} \tag{19}$$

Substituting (3) and (4) into the first equation of (5) (which gives the definition of K_Y) and using the quality indices $Q_{\mathcal{C}} = Q - Q_{\mathcal{M}}$ and the last equation of (5) (which givens the definition of $Q_{\mathcal{M}}$), we obtain

$$\begin{aligned} K_Y &= \eta\left(\sum_{i\in\mathcal{C}}x_i + \sum_{i\in\mathcal{M}}x_i\right) \\ &= \eta\left(\frac{\alpha}{r\eta}\right)^{\frac{1}{1-\alpha}} L\left(\sum_{i\in\mathcal{C}}\lambda^{\frac{\alpha}{1-\alpha}M_i} + \alpha^{\frac{1}{1-\alpha}}\sum_{i\in\mathcal{M}}\lambda^{\frac{\alpha}{1-\alpha}M_i}\right) \\ &= \eta\left(\frac{\alpha}{r\eta}\right)^{\frac{1}{1-\alpha}} L\,(Q - Q_{\mathcal{M}} + \alpha^{\frac{1}{1-\alpha}}Q_{\mathcal{M}}). \end{aligned} \tag{20}$$

Eliminating r by using Y in (19) and K_Y in (20) yields

$$Y = \eta^{-\alpha}L^{1-\alpha}K_Y^{\alpha}\frac{Q - Q_{\mathcal{M}} + \alpha^{\frac{\alpha}{1-\alpha}}Q_{\mathcal{M}}}{(Q - Q_{\mathcal{M}} + \alpha^{\frac{1}{1-\alpha}}Q_{\mathcal{M}})^{\alpha}}. \tag{21}$$

Thus, by using function Ψ, we obtain the aggregate GDP level Y, denoted as (6).

Solving (20) with respect to r provides the following:

$$r = \alpha\,\eta^{-\alpha}K_Y^{\alpha-1}L^{1-\alpha}(Q - Q_{\mathcal{C}} + \alpha^{\frac{1}{1-\alpha}}Q_{\mathcal{M}})^{1-\alpha} \tag{22}$$

Using function Φ, the above equation can be written as (8).

By using $\pi_{(i,M_i)} = p_{(i,M_i)}x_{(i,M_i)} - r\eta x_{(i,M_i)}$ and $x_i = (\alpha^2/(r\eta))^{\frac{1}{1-\alpha}}L\lambda^{\frac{\alpha}{1-\alpha}M_i}$ derived in (5), Y derived in (21), and r derived in (22), the profit of the ith sector monopoly firm with a patent of quality M_i is obtained as follows:

$$\begin{aligned} \pi_{(i,M_i)} &= \left(\frac{1}{\alpha} - 1\right) r\eta x_{(i,M_i)} \\ &= (1-\alpha)\alpha^{\frac{2}{1-\alpha}}\eta^{-\alpha}K_Y^{\alpha}L^{1-\alpha}(Q - Q_{\mathcal{M}} + \alpha^{\frac{1}{1-\alpha}}Q_{\mathcal{M}})^{-\alpha}\lambda^{\frac{\alpha}{1-\alpha}M_i} \\ &= (1-\alpha)\alpha^{\frac{1}{1-\alpha}}\frac{Y}{Q - Q_{\mathcal{M}} + \alpha^{\frac{\alpha}{1-\alpha}}Q_{\mathcal{M}}}\lambda^{\frac{\alpha}{1-\alpha}M_i}. \end{aligned} \tag{23}$$

B. Condition for the Existence of an Equilibrium without R&D

We first give the order of some obtained critical values of capital stock.

Proposition The order of the critical values of capital stock is given as follows:

$$\frac{1}{\delta} < \frac{1}{\delta\left(1-\alpha^{\frac{1}{1-\alpha}}\right)}(\equiv k^{--}) < \frac{1}{\delta(1-\alpha)}(\equiv k^{-}) < \frac{1}{\delta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}(\equiv k^{+})$$

Proof) $0<\alpha<1$ derives $1 < \frac{1}{1-\alpha^{\frac{1}{1-\alpha}}}$, and $\alpha^{\frac{\alpha}{1-\alpha}} < 1$ (equivalently $\alpha^{\frac{1}{1-\alpha}} < \alpha$), which implies $1 < \frac{1}{1-\alpha^{\frac{1}{1-\alpha}}} < \frac{1}{1-\alpha} < \frac{1}{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}$. From these results, it follows that $1/\delta < k^{--} < k^{-} < k^{+}$ (Q.E.D.).

On Condition (R), if $RHS > LHS$ holds at point $k_A = 0$, there exists an equilibrium without R&D $k_A \equiv k_A^0 (= 0)$. Under the assumption that $\mu(0) = 0$, the above condition is translated into $k < k^+$. This equation shows that if $k \leq k^+$, then $k_A = 0 (\equiv k_A^0)$ is the solution of (R). In other words, a small capital stock may bring about an equilibrium without R&D. It also implies that if $k > k^+$, then k_A^0 cannot be a solution for (R). Therefore, a sufficiently large capital stock engenders positive R&D activities. Stated differently, k^+ is the upper bound of the existence of the equilibrium without R&D.

C. Arbitrage Equation of R&D

Under (P), both sides of (11) chnage into the following:

$$LHS(k_A) = \begin{cases} \delta\varphi & \text{for} \quad k_A < \frac{1}{\delta} \\ \frac{\varphi}{k_A} & \text{for} \quad k_A > \frac{1}{\delta} \end{cases}, \tag{24}$$

$$RHS(k_A) = \begin{cases} \frac{\alpha\left(1-\delta k_A + \alpha^{\frac{1}{1-\alpha}}\delta k_A\right)}{k-k_A} & \text{for} \quad k_A < \frac{1}{\delta} \\ \frac{\alpha^{\frac{2-\alpha}{1-\alpha}}}{k-k_A} & \text{for} \quad k_A > \frac{1}{\delta} \end{cases}. \tag{25}$$

First, the condition that k_A^0 is in equilibrium is necessary to hold $k \leq k^+$ from Appendix A. The solution for $LHS(k_A) = RHS(k_A)$ is considered next. If

$k_A < \frac{1}{\delta}$, then $LHS(k_A) = RHS(k_A)$ becomes

$$\delta\varphi = \delta(1-\alpha)\alpha^{\frac{1}{1-\alpha}} = \frac{\alpha(1 - \delta k_A + \alpha^{\frac{1}{1-\alpha}}\delta k_A)}{k - k_A}. \tag{26}$$

Solving this equation, we obtain the following solution:

$$\text{Small R\&D Activities:} \quad k_A = \frac{1 - \delta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} k}{\delta(1 - \alpha^{\frac{\alpha}{1-\alpha}})} \equiv \bar{k}_A.$$

For $\bar{k}_A$ to be the solution for (26), it is necessary to fulfill the conditions of $0 \leq k_A < \frac{1}{\delta}$. Solving this condition for k, the necessary condition is obtained as follows:

$$k^- \leq k \leq k^+ \quad \Rightarrow \quad \text{Small R\&D Activities: } \bar{k}_A, \tag{27}$$

where $k^- \equiv \frac{1}{\delta(1-\alpha)}$.

If $k_A \leq \frac{1}{\delta}$, then $LHS(k_A) = RHS(k_A)$ becomes

$$\frac{1-\alpha}{k_A} = \frac{\alpha}{k - k_A}. \tag{28}$$

Solving this equation yields the following solution:

$$\text{Large R\&D Activities:} \quad k_A = (1-\alpha)k \equiv k_A^*.$$

$(1-\alpha)k > \frac{1}{\delta}$ is necessary for k_A^* to be the solution for (28). Solving this condition, we obtain

$$k > k^- \quad \Rightarrow \quad \text{Large R\&D Activities : } k_A^*. \tag{29}$$

From $k < k^+$, (27) and (29), the equilibrium R&D capital investments are divided into the following set of equilibria: when the economy accumulates more capital than k^-, an equilibrium with positive investment for R&D exists. In other words, k^- is the upper limit of existence of an equilibrium without R&D. Therefore an economy has a set of an equilibrium without R&D $\mathcal{K}_A^N = \{k_A^0\}$ when $k < k^-$. When the capital stock is in interval $[k^-, k^+]$, the economy has multiple equilibria: $\mathcal{K}_A^M = \{k_A^0, \bar{k}_A, k_A^*\}$. When the capital stock is larger than k^+, both equilibria – k_A^0 and $\bar{k}_A$ – vanish and a unique equilibrium $k_A = k_A^*$ exists. We represent this case as $\mathcal{K}_A^L$. These sets are summarized as follows.

$$k < k^- \Rightarrow \text{Set of an Equilibrium without R\&D:} \quad \mathcal{K}_A^N$$
$$k^- \leq k \leq k^+ \Rightarrow \text{Multiple Equilibria Set:} \quad \mathcal{K}_A^M$$
$$k^+ < k \Rightarrow \text{Set of an Equilibrium with Large R\&D:} \quad \mathcal{K}_A^L$$

Table 1 lists these equilibria. As capital accumulates, the economy changes the equilibrium set. At the first stage ($k < k^-$), the set is a unique equilibrium without R&D ($\mathcal{K}_A^0$). At the second stage ($k^- < k < k^+$), it is a set with multiple equilibria ($\mathcal{K}_A^M$). The last stage ($k^+ < k$) of the economy has a set with a unique equilibrium with a large positive R&D investment ($\mathcal{K}_A^L$). It should be noted that $\bar{k}_A$ coincides with k_A^0 at $k_t = k^+$ and k_A^* at $k_t = k^-$.

From (24), LHS is constant when $k_A \in (0, \frac{1}{\delta})$, decreases monotonically when $k_A \geq \frac{1}{\delta}$, and converges to 0 to the limit of an infinite amount of capital. Differentiating RHS with respect to k_A, we easily verify the properties as follows. When the capital stock is small ($k < k^{--}$), RHS has increasing returns ($\frac{\partial RHS}{\partial k_A} > 0$). When the capital stock is large ($k > k^{--}$), RHS decreases in $k_A < \frac{1}{\delta}$, and increases in $\frac{1}{\delta} < k_A < k$. The second order derivative of RHS is always positive. These points are summarized as follows:

$$\frac{\partial RHS}{\partial k_A} > 0, \quad \text{for} \quad k < k^{--} \quad k_A \in (0, k),$$
$$\frac{\partial RHS}{\partial k_A} \left\{ \begin{array}{c} < \\ > \end{array} \right\} 0, \quad \text{for} \quad k > k^{--}, \quad k_A \in \left\{ \begin{array}{c} (0, \frac{1}{\delta}) \\ (\frac{1}{\delta}, k) \end{array} \right. ,$$
$$\frac{\partial RHS^2}{\partial^2 k_A} > 0, \quad \text{for} \quad k_A \in (0, k).$$

In light of these properties, Figure 2 depicts the LHS and RHS contours.

References

[1] Abramovitz, M. and P. A. David (1973) "Reinterpreting Economic Growth: Parables and Realities", American Economic Review, Vol. 63, No. 2, pp. 428-439.

[2] Aghion, P. and P. Howitt (1998) Endogenous Growth Theory, Cambridge, MA: MIT Press.

[3] Barro, R. J. and X. Sala-i-Martin (1995) Economic Growth, New York: McGraw-Hill.

[4] Brezis, E., P. Krugman and D. Tsiddon (2001) "Leapfrogging in International Competition: A Theory of Cycles in National Technological Leadership", American Economic Review, Vol.83, pp.1211-1219.

[5] Easterly, W. (1994) "Economic Stagnation, Fixed Factors and Policy Thresholds", Journal of Monetary Economics, Vol. 33, pp. 525-557.

[6] Galor, O. and O. Moav (2004) "From Physical to Human Capital Accumulation: Inequality and the Process of Development", Review of Economic Studies, Vol. 71, No. 4, pp. 1001-1026.

[7] Grossman, G. M. and E. Helpman (1991) Innovation and Growth in the Global Economy, Cambridge, MA: MIT Press.

[8] Hayami, Y., and J. Ogasawara (1999) "Changes in the Sources of Modern Growth: Japan Compared with the United States", Journal of Japanese and International Economies, Vol. 13, pp. 1-21.

[9] Irmen, A. (2005) "Extensive and Intensive Growth in a Neoclassical Framework", Journal of Economic Dynamics and Control, Vol. 29, pp.1427-1448.

[10] Jones, C. I. (1995) "R&D-Based Models of Economic Growth", Journal of Political Economy, Vol. 103, pp. 759-784.

[11] Krugman, P. (1991) "History versus Expectations", The Quarterly Journal of Economics, Vol. 106, No. 2, pp. 651-67.

[12] Lucas, R. E. Jr. (1988) "On the Mechanism of Economic Development", Journal of Monetary Economics, Vol. 22, No. 1, pp. 3-42.

[13] ——- (1993) "Making a Miracle", Econometrica, Vol. 61 No. 2, pp. 251-72.

[14] Matsuyama, K. (1999) "Growing Through Cycles", Econometrica, Vol. 67, No. 2, pp. 335-347.

[15] Quah, D. (1996) "Empirics for Economic Growth and Convergence", European Economic Review, Vol. 40, No. 6, pp. 1353-1375.

[16] ——- (1997) "Empirics for Growth and Distribution: Polarization, Stratification, and Convergence Clubs", Journal of Economic Growth, Vol. 2, No. 1, pp. 27-59.

[17] Romer, P. M. (1990). "Endogenous Technological Change", Journal of Political Economy, Vol. 98 No. 5, pp. S71-S102.

[18] Shell, K. (1967) "A Model of Inventive Activity and Capital Accumulation", In: K. Shell, ed., Essays on the Theory of Optimal Economic Growth, Cambridge MA: MIT Press.

[19] ——- (1973). Inventive Activity, Industrial Organization and Economic Growth, in J. A. Mirrlees and N. H. Stern, ed., Models of Economic Growth, London: MacMillan.

[20] Zilibotti, F. (1995) "A Rostovian Model of Endogenous Growth and Underdevelopment Traps", European Economic Review, Vol. 39, pp. 1569-1602.

Table 1: Equilibrium Capital Allocation and the Equilibrium Set

Equilibrium Type (Condition)	No R&D k_A^0 $(k < k^-)$	Small R&D $\bar{k}_A$ $(k^- < k < k^+)$	Large R&D k_A^* $(k > k^+)$
No R&D Equilibrium	○		
Multiple Equilibria	○	○	○
R&D Equilibrium			○

○: Equilibrium

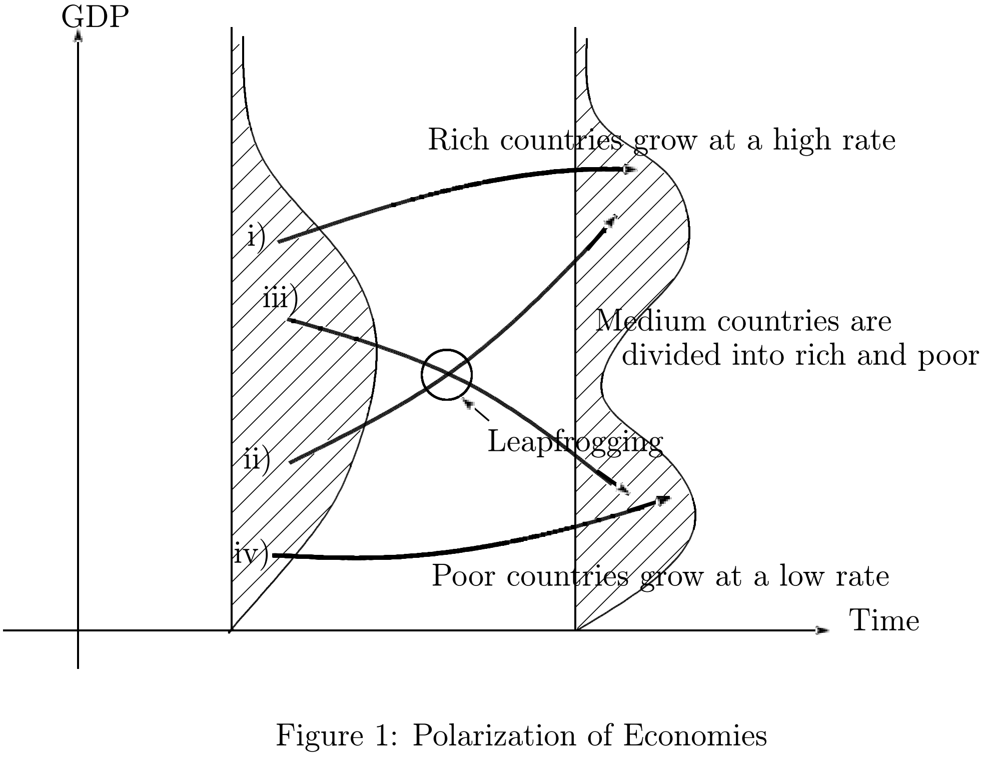
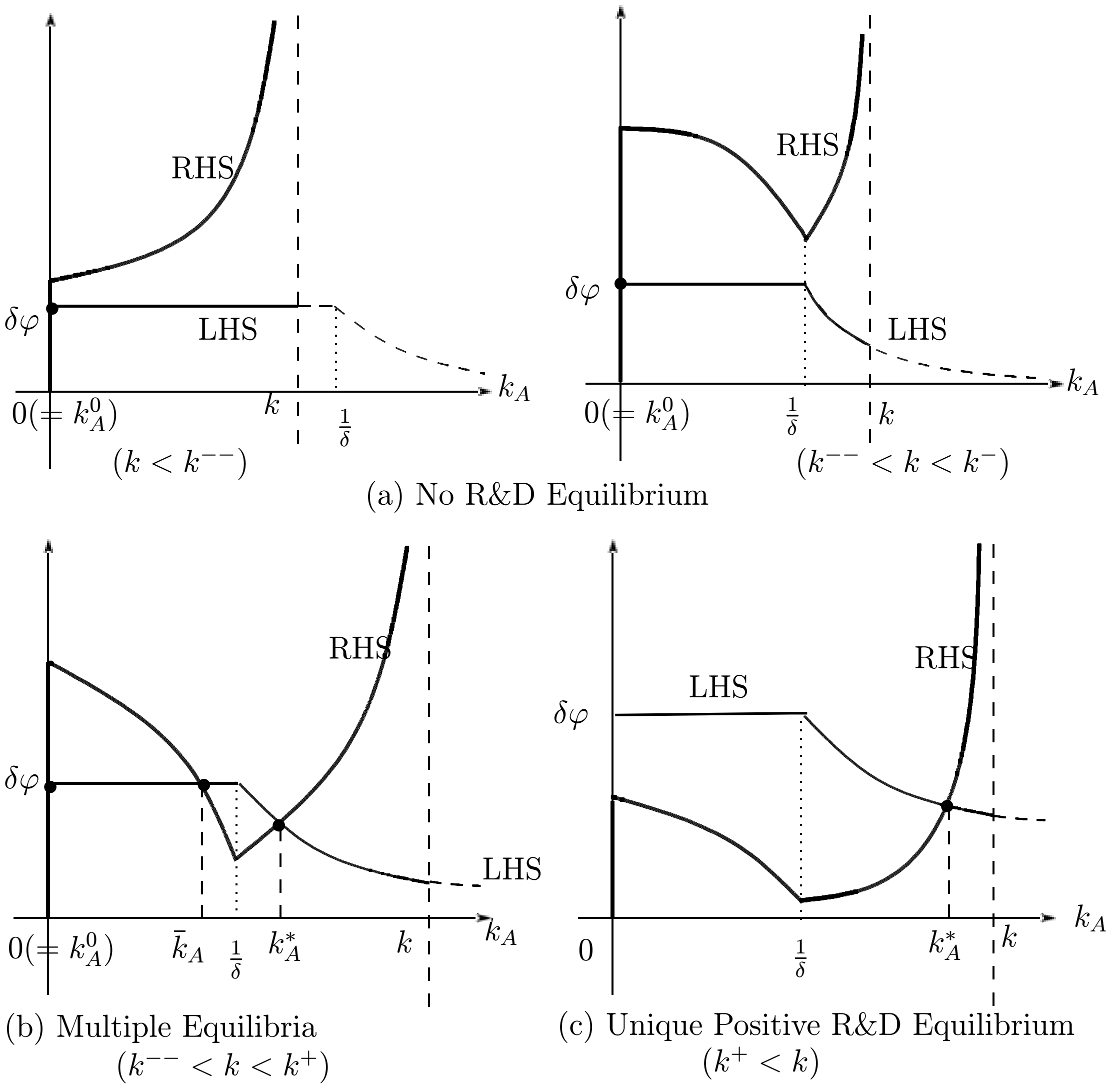
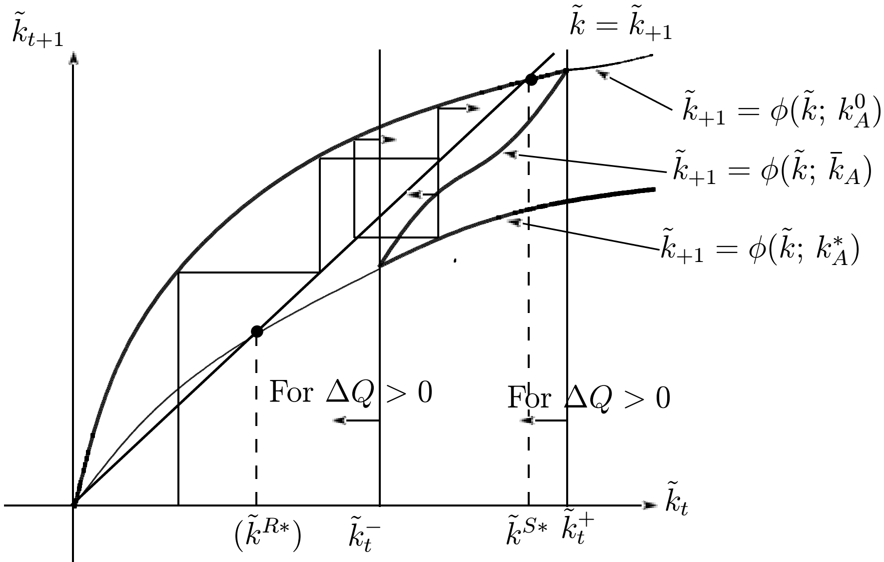


Figure 1: Polarization of Economies

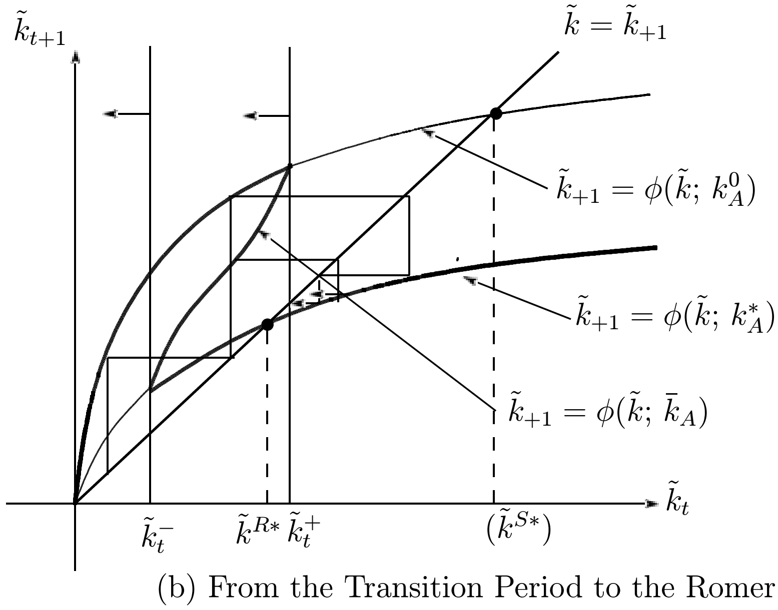


• : Equilibrium Capital Accumulation

Figure 2: Equilibrium Capital Allocation



(a) From the Solow Regime to the Transition Period



(b) From the Transition Period to the Romer Regime

Figure 3: Dynamics of $\tilde{k}_t$ and Regime Switch